1 Problem 1

This is a follow-up of Exercise 1. The idea is to go back to Exercise 1 and reinterpret in terms of what you have learned later.

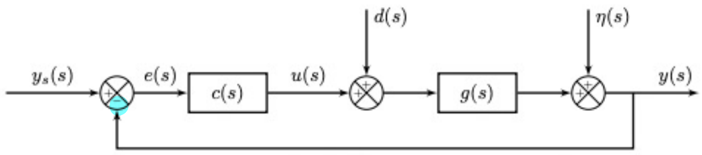


Figure 1: Controlled System

In the block diagram we have $d = \Delta T_F$, $y = \Delta T$ and $u = \Delta Q/(F c_P)$. The transfer function is given by

$$g(s) = \frac{e^{-100s}}{20s+1} \tag{1}$$

1.1 P-control with $c(s) = K_c$

Derive the closed-loop transfer functions from d to u and y:

$$y = h_1(s)d$$
$$u = h_2(s)d$$

What are the steady-state values of y and u when $d = 1$ (unit step) as a function of K_c, that is, what is $h_1(0)$ and $h_2(0)$? What are the values for $K_c = 0.5$? Does this agree with the simulations in Exercise 1? Why is there a steady-state offset, that is, why does not $y(t)$ return back to 0?

CL: $T_{CL} = \dfrac{\text{dir loop}}{1 + \text{loop}}$

$$\Rightarrow h_1 = \frac{g}{1+g\cdot c} = \frac{g}{1+K_c g} = \frac{e^{-100s}/20s+1}{1+K_c e^{-100s}/20s+1} = \frac{e^{-100s}}{20s+1+K_c e^{-100s}}$$

$$\underline{\underline{h_1 = \frac{e^{-100s}}{20s+1+K_c e^{-100s}}}}$$

$$h_2 = \frac{-g\cdot c}{1+g\cdot c} = \frac{-e^{-100s}/20s+1}{1+K_c e^{-100s}/20s+1} = \frac{-K_c e^{-100s}}{20s+1+K_c e^{-100s}}$$

$$\underline{\underline{h_2 = \frac{-K_c e^{-100s}}{20s+1+K_c e^{-100s}}}}$$

SS-vals:

When d is a unit step, $\mathcal{L}(d) = \frac{1}{s}$, then, using the final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\cdot T(s) = \lim_{s\to 0} s\cdot h_1(s)\cdot\frac{1}{s} = \lim_{s\to 0} h_1(s) = \frac{1}{1+K_c} = \frac{1}{1+0{,}5}$$

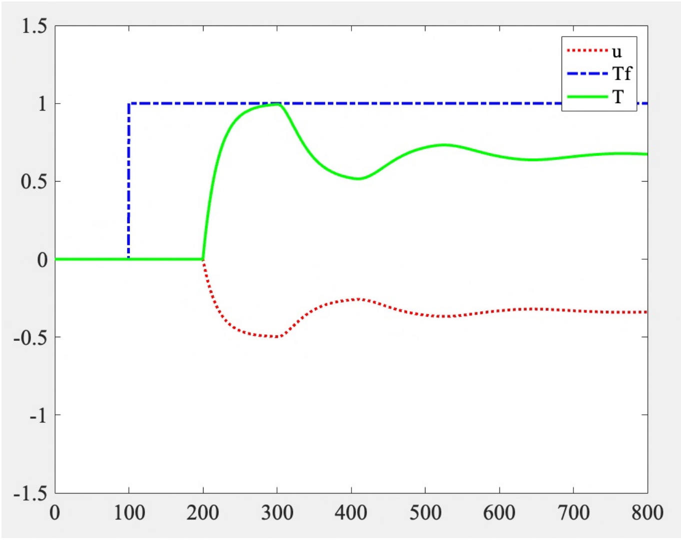
$$\Rightarrow \underline{\underline{y(\infty) = 0{,}667}}$$

Similarily:

$$\lim_{t\to\infty} u(t) = \lim_{s\to 0} h_2(s) = \frac{-K_c}{1+K_c} = \frac{-0{,}5}{1+0{,}5}$$

$$\underline{\underline{u(\infty) = -0{,}333}}$$

This looks accurate compared to 2,3-2 in exercise 1.



We have a steady-state offset due to a lack of integral action.
We need $C(s) \to \infty$ when $s=0$

1.2 SIMC PI-control

Derive the SIMC PI-controller (K_c, τ_I) for the choices $\tau_c = 100$, $\tau_c = 50$ and $\tau_c = 0$. What is the steady-state value of y and u when $d = 1$ (unit step)? Plot the responses for $y(t)$ and $u(t)$ to a step in d (see solution to Exercise 1).

From ex 1: $k=1$, $\tau_1 = 20$

$$K_c = \frac{1}{k} \cdot \frac{\tau_1}{\tau_c + \theta} \quad , \quad \tau_I = \min(\tau_1, 4(\tau_c+\theta))$$

$\underline{\tau_c = 100:}$

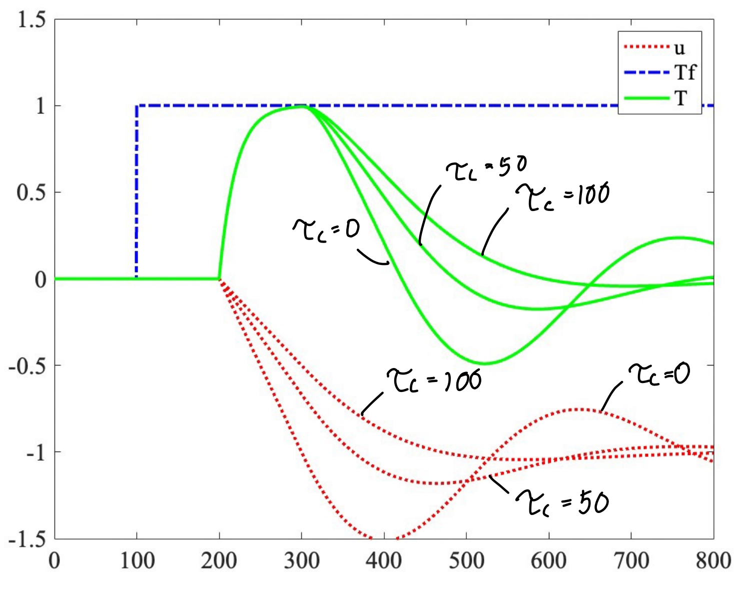
$$K_c = \frac{1}{1} \cdot \frac{20}{100+100} = 0,1$$

$$\tau_I = \min(20, 400) = 20$$

$\underline{\tau_c = 50:}$

$$K_c = \frac{1}{1} \cdot \frac{20}{50+100} = 0,133$$

$$\tau_I = \min(20, 600) = 20$$



$\underline{\tau_c = 0:}$

$$K_c = \frac{1}{1} \cdot \frac{20}{100} = 0,2$$

$$\tau_I = \min(20, 400) = 20$$

Now, having PI, the transfer function is $C(s) = K_c \frac{\tau_I s+1}{\tau_I s}$

$$\Rightarrow h_1 = \frac{g}{1+g\cdot c} = \frac{e^{-100s}/20s+1}{1+\frac{e^{-100s}}{20s+1}\cdot K_c \cdot \frac{\tau_I s+1}{\tau_I s}} = \frac{e^{-100s}}{(20s+1)+K_c e^{-100s}\cdot\frac{\tau_I s+1}{\tau_I s}}$$

$$h_1 = \frac{\tau_I s e^{-100s}}{\tau_I s(20s+1)+K_c e^{-100s}\cdot(\tau_I s+1)}$$

We can see that $y(\infty) = h_1(0) = 0 \;\forall\, \tau_c$ from the expression

$$h_2 = \frac{-g\cdot C}{1+g\cdot C} = \frac{-K_c\cdot\frac{e^{-100s}}{20s+1}\cdot\left(\frac{\tau_I s+1}{\tau_I s}\right)}{1+K_c\cdot\frac{e^{-100s}}{20s+1}\left(\frac{\tau_I s+1}{\tau_I s}\right)} = \frac{-K_c\cdot e^{-100s}(\tau_I s+1)}{\tau_I s(20s+1)+K_c e^{-100s}(\tau_I s+1)}$$

$$u(\infty) = h_2(0) = \frac{-K_c}{K_c} = -1$$

$$\Rightarrow u(\infty) = -1 \;\forall\, \tau_c$$

1.3 Ziegler-Nichols PI

In Exercise 1 we found that the "critical" or "ultimate" gain with P-control is $K_u = 1.13$. Use the simulations from Exercise 1 to derive Ziegler-Nichols PI-settings.

Adding a piece of code to the script from exercise 1:

```
% Finner toppene og perioden til exercise 7 oppgave 1.3
bigboi1 = 0;
peaktime1 = 0;
bigboi2 = 0;
peaktime2 = 0;
for i = 1:size(T)
    if time(i) > 500 & time(i) < 600
        if T(i) > bigboi1
            bigboi1 = T(i);
            peaktime1 = time(i);
        end
    elseif time(i) > 700
        if T(i) > bigboi2
            bigboi2 = T(i);
            peaktime2 = time(i);
        end
    end
end

disp('Perioden er')
disp(peaktime2 - peaktime1)
```

Looking at graph from exercise 1

$\Rightarrow P = 234{,}35\,s$

Using the rules for PI

ess information obtain PID settings:

Table 11.4 Controller Settings based on the Continuous Cycling Method

Ziegler-Nichols	K_c	τ_I	τ_D
P	$0.5K_{cu}$	—	—
PI	$0.45K_{cu}$	$P_u/1.2$	—
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$

PID is for ideal form

We get

$K_c = 0{,}45\cdot 1{,}13 = 0{,}5085$

$\tau_I = \frac{234{,}35}{1{,}2} = 195{,}29$

1.4 Smith predictor: time-delay compensation

What is the Smith Predictor controller for this process (with τ_c as a parameter)? Show how it can be realized in a block diagram.

Note that setting $\tau_c = 0$ would give "ideal control", but in practise this controller would not be robust.

The closed loop transfer function:

$$T_{CL} = \frac{C \cdot g}{1 + C \cdot g} \quad \text{, we must find } c\text{, know } g = \frac{e^{-\theta s}}{20s+1} \quad , \theta = 100$$

* We are looking to find a C_{SP}, so we get a desired set-point response

Then, setting the CL-transfer functions equal to the wanted first order response

$$T_{CL} = \frac{y}{y_s} \Rightarrow \frac{g \cdot C_{SP}}{1 + g \cdot C_{SP}} = \frac{e^{-\theta s}}{\tau_c s + 1} \quad / \cdot \frac{1}{n}$$

$$\frac{1}{g \cdot C_{SP}} + 1 = \frac{\tau_c s + 1}{e^{-\theta s}}$$

$$\frac{1}{g \cdot C_{SP}} = \frac{\tau_c s + 1 - e^{-\theta s}}{e^{-\theta s}}$$

$$\Rightarrow C_{SP} = \frac{1}{g} \cdot \frac{e^{-\theta s}}{\tau_c s + 1 - e^{-\theta s}} \quad / \text{insert } g$$

$$C_{SP} = \frac{20s+1}{\cancel{e^{-\theta s}}} \cdot \frac{\cancel{e^{-\theta s}}}{\tau_c s + 1 - e^{-\theta s}}$$

$$\underline{\underline{C_{SP} = \frac{20s+1}{\tau_c s + 1 - e^{-\theta s}}}}$$

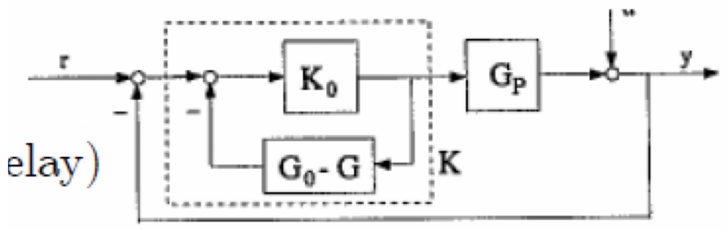
We want a structure like this, where K is the inner transfer loop

K_o is the primary controller

G_o is the model without delay

G is the model with delay

G_p is the actual plant



(a)

$\Rightarrow$ Assuming a perfect model, $G_p = \dfrac{e^{-\theta s}}{20s+1} = G$

$$G = \frac{e^{-\theta s}}{20s+1}, \quad G_0 = \frac{1}{20s+1}$$

$$G_0 - G = \frac{1}{20s+1}\left(1 - e^{-\theta s}\right)$$

We want: K to be C_{SP}. For the closed loop:

$$K = \frac{K_0}{1 + K_0(G_0 - G)} = C_{SP} = \frac{20s+1}{\tau_c s + 1 - e^{-\theta s}}$$

$$\frac{K_0}{1 + \frac{K_0}{20s+1}\left(1 - e^{-\theta s}\right)} = \frac{20s+1}{\tau_c s + 1 - e^{-\theta s}}$$

$$\frac{K_0\left(\cancel{20s+1}\right)}{20s+1 + K_0\left(1 - e^{-\theta s}\right)} = \frac{\cancel{20s+1}}{\tau_c s + 1 - e^{-\theta s}}$$

$$\frac{K_0}{20s+1 + K_0\left(1 - e^{-\theta s}\right)} = \frac{1}{\tau_c s + 1 - e^{-\theta s}}$$

$$K_0\left(\tau_c s + 1 - e^{-\theta s}\right) = 20s+1 + K_0\left(1 - e^{-\theta s}\right)$$

$$K_0 \cdot \tau_c s = 20s+1$$

$$K_0 = \frac{20s+1}{\tau_c s}$$

We then get the following block diagram