Exercise 2

1 Distillation Case Study

Distillation is a method of separating liquid mixtures by means of partial evaporation. The volatility α of a components determines how enriched the liquid or vapour phase will be in that component. Components with high volatility will enrich the vapour phase and components with low volatility will enrich the liquid phase. A distillation column uses counter current flow of vapour and liquid and several distillation stages to achieve a high purity product.

1.1 Control Structure

Task 1: A typical distillation column with valves and measurements is given in Figure 1. Please do the following:

(a) Define the control objective.

(b) Classify the variables into: Control Variables, CVs (y), Manipulated Variables, MVs (u) and disturbance variables DVs (d).

(c) Fill in the process matrix. (Table with CVs along the rows and MVs along the columns, where the elements are filled 0, +, -, (+), (-) representing qualitative relation between them).

(d) Suggest a control structure for the column.

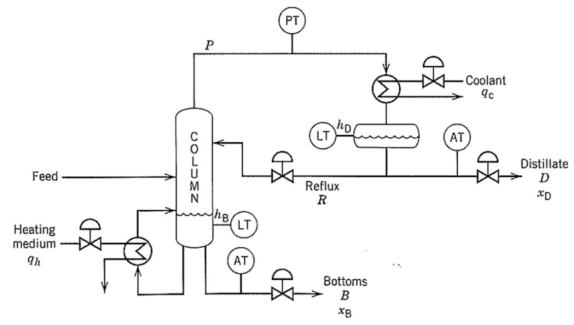


Figure 1: Distillation column setup. x_D and x_B, respectively, are the mole fraction of light component at the top and bottom of the column

a) We want to keep X_D constant (and as high as possible I assume) as well as keeping X_B low

b) CV: h_B, h_D, X_D, X_B, P

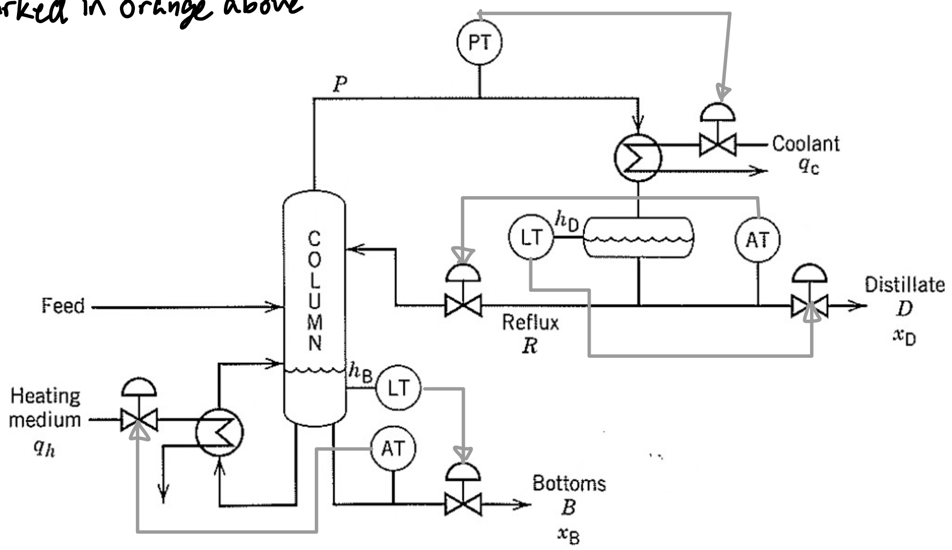
MV: R, B, D, q_c, q_h

DV: F (feed), Z_F

c)

MV \ CV	h_B	h_D	X_D	X_B	P
R	+	−	+	(+)	(-)
B	−	0	0	0	(-)
D	0	−	0	0	(-)
q_c	0	+	0	0	−
q_h	−	+	−	−	+

d) Marked in orange above



1.2 Modelling

In rest of this case study we are going to investigate a simple 4 stage (considering reboiler and total condenser) distillation column. See Figure 2 for a sketch of the process. Consider the following assumptions and parameters:

- Constant relative volatility.
- Constant pressure and no pressure drop.
- Constant molar overflow.
- Top stage is a total condenser (i.e. all vapour is condensed and there is only liquid flow out)
- The feed is saturated liquid.

Parameters	
Number of stages	4
Number of components	2
Relative volatility, α	4.78

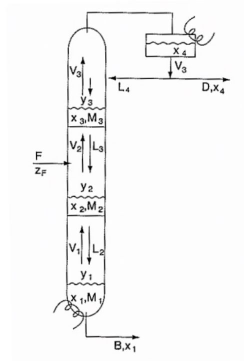


Figure 2: Sketch of distillation column model

Task 2: Using the nomenclature in Figure 2, and the assumptions and parameters given above, please write a dynamic model for the distillation column (Hint: write the dynamic component balances for all stages). Use the following liquid vapour equilibrium relation:

$$y = \frac{\alpha x}{1 + (\alpha - 1)x}$$

where y and x is the light component molar fraction in the vapour and liquid, respectively.

Mass = mole balances

$$\frac{dM_1}{dt} = L_2 - V_1 - B$$

$$\frac{dM_2}{dt} = V_1 + L_3 + F - V_2 - L_2$$

$$\frac{dM_3}{dt} = V_2 + L_4 - L_3 - V_3$$

$$dM_4 = V_3 - L_4 - D$$

In order to have mass/mole equilibrium inside the column:

$$L_4 = L_3$$

$$L_3 + F = L_2$$

$$V_1 = V_2$$

$$V_2 = V_3$$

Component balances:

$$\frac{d(M_1 x_1)}{dt} = L_2 x_2 - V_1 y_1 - B x_1$$

$$\frac{d(M_2 x_2)}{dt} = V_1 y_1 + L_3 x_3 + F z_F - V_2 y_2 - L_2 x_2$$

$$\frac{d(M_3 x_3)}{dt} = V_2 y_2 + L_4 x_4 - V_3 y_3 - L_3 x_3$$

$$\frac{d(M_4 x_4)}{dt} = V_3 y_3 - L_4 x_4 - D x_4$$

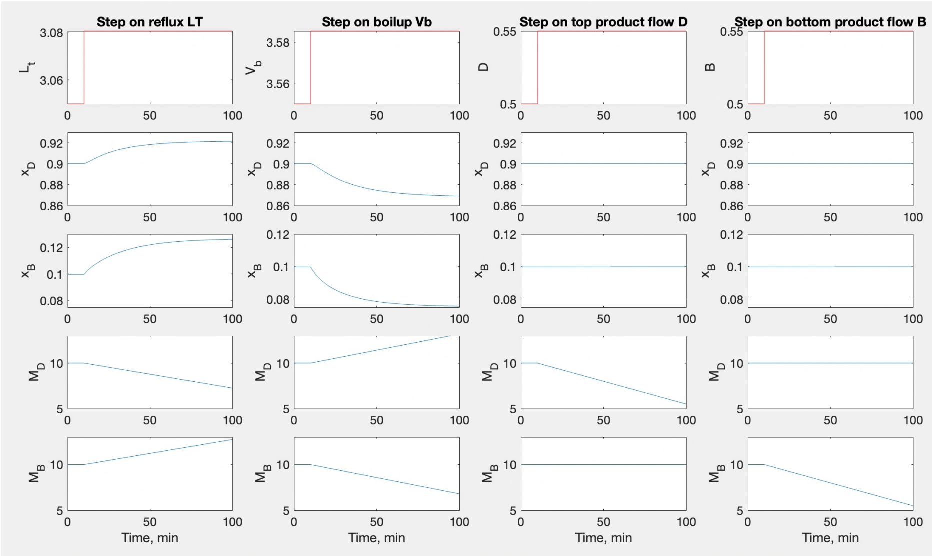
For each vaporization step, VLE gives:

$$y_1 = \frac{\alpha x_1}{1 + (\alpha - 1)x_1}$$

$$y_2 = \frac{\alpha x_2}{1 + (\alpha - 1)x_2}$$

$$y_3 = \frac{\alpha x_3}{1 + (\alpha - 1)x_3}$$

Task 3: Here we consider the loops are all open (no control). This means that $Kc = Kd = Kb = 0$. In the script `task3.m` we apply a 1% step increase in the reflux Lt and in the boilup Vb, and a 10% step increase in the distillate (D) and bottom (B) flows. Run the script and then compare the responses you will see with the process matrix you derived in Task 1. Were your predictions correct? Together with this exercise we provide a sheet with some initial plots. Fill in by hand the remaining plots in Figure 4 according to what you see from the simulations.



not included

MV \ CV	M_B ~~h_B~~	M_D ~~h_D~~	X_D	X_B	P
L_T ~~R~~	+	−	+	(+)	(−)
B	−	0	0	0	(−)
D	0	−	0	0	(−)
~~q_c~~	~~0~~	~~+~~	~~0~~	~~0~~	~~−~~
V_b ~~q_h~~	−	+	−	−	+

It appers that I'm right

Increasing reflux will tap the liquid in the destillete tank, and increase in the bottom of the column.

The mole fractions X_D, X_B increase due to the top product increasing its purity, however, more of the light component will be lost in the bottom of the column, which is why X_B increases

Increasing B or D will drein their respective liquid levels

Increasing V_b causes more liquid to evaporate in the bottom $\Rightarrow h_B$ decreases

but more vapor will cool down in the top $\Rightarrow h_D$ increases

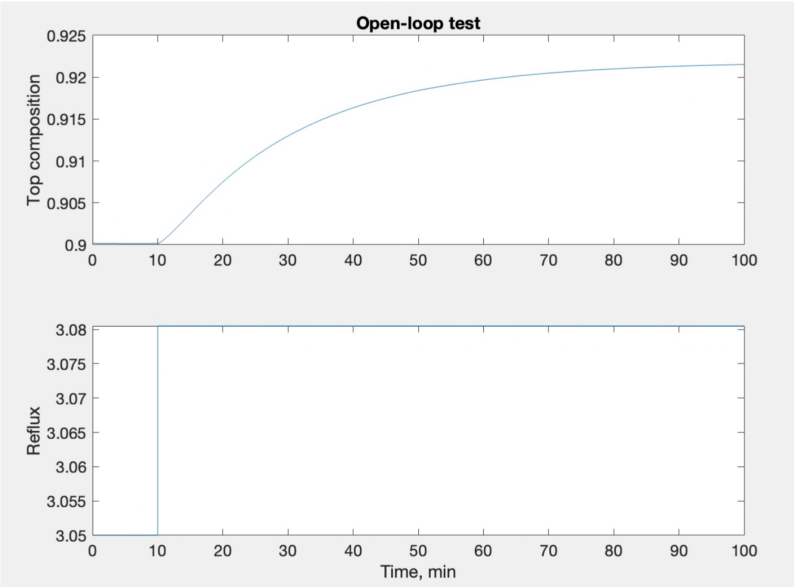
More light component evaporates from the bottom than heavy component $\Rightarrow X_B$ decreases

The "new" vapor is less pure than the distillate before the heat increase $\Rightarrow X_D$ decreases.

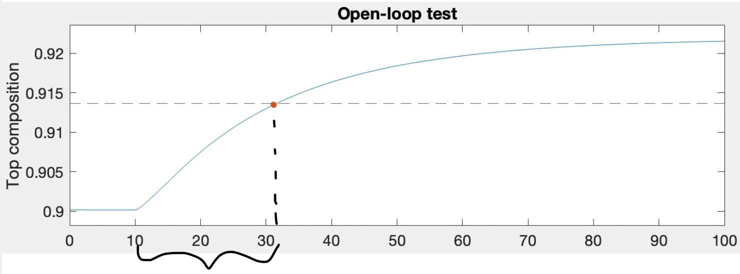
Task 4: In this task we must close the level control loops. For this task we are going to use the files `task4_steptest.m` and `task4_controleval.m`. Before starting, be sure $Kb = Kd = -10$ in the Matlab scripts. We want to control the top composition (x_D) using the reflux (Lt). Please design a PI-controller to accomplish this task by following the *PI tuning* procedure presented below.
Should the gain of the controller be negative or positive? Please explain.

- It should be positive, increasing $R(L_t)$ increases x_D $\Rightarrow$ The gain should be positive for a feedback controller.

1. Apply a step change on Lt and check how the top composition changes in open-loop ($Kc = 0$). (Hint: this can be done by running the script `task4_steptest.m`. The reflux is increased with the values of the variable `LT_step`.)



After messing with the code, I was able to find the closest value to $0{,}63 \cdot \Delta y$



$\approx 21{,}16 \text{ min} = \theta + \tau$

2. Using the plot obtained, find a 1st order transfer function model $\left(G(s) = k\frac{e^{-\theta s}}{\tau s+1}\right)$ from u to y (where $y \equiv x_D$). (Hint, the steady-state gain is $k = \frac{\Delta y}{\Delta u}$ at steady state. The time constant τ is approximately the time it takes, after the delay, for y to reach 63% of its total variation value. *Note: remember to substract the step time, i.e. the value of LT_ change_ time in the Matlab script*). Make a plot with responses and mark the key information as seen in Figure 3.

From the matlab code:

$\Delta y = 0{,}021366$

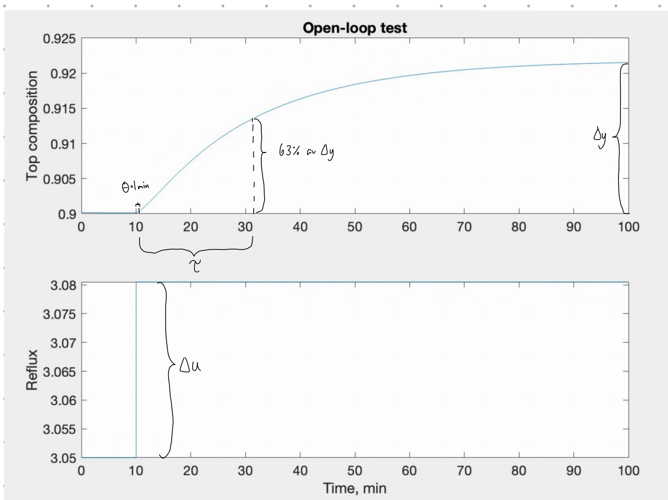
$\Delta u = \underbrace{3{,}05}_{\text{found in code}} \cdot 0{,}01 = 0{,}0305$

$k = \frac{\Delta y}{\Delta u} = 0{,}7$

In 3. it is given that $\theta = 1$ min

$\Rightarrow \tau = 20{,}16$ min

$\Rightarrow \quad G(s) = 0{,}7 \dfrac{e^{-s}}{20{,}16 s + 1}$



3. Tune a PI controller using SIMC rule (Eq. 1 - *must be memorized!*. Try $\tau_c = 3$ min, $\tau_c = 1$ min and $\tau_c = 0.5$ min and simulate for disturbances and setpoint changes. Print the results and give the parameters obtained. *Notice that a measurement delay of 1 min has been included in the feedback loop, so $\theta = 1$ min.*

The SIMC (simple internal model control) rules for tuning a PI-controller are given in Eq. 1. We will come back to it in lecture 6 and exercise 6.

$$K_c = \frac{1}{k}\frac{\tau}{\tau_c + \theta} \tag{1a}$$

$$\tau_I = \min(\tau_c + \theta) \tag{1b}$$

where, K_c is the controller proportional gain, τ_I is the integral time, k is the process steady-state gain, τ process time constant, τ_c is the closed loop time constant and θ is the delay.

For 3 min

$$K_c = \frac{1}{0{,}7} \cdot \frac{20{,}16}{3+1} = 7{,}2$$

$$\tau_I = \min\left(20{,}16, \underbrace{4\cdot(3+1)}_{=16}\right) = 16$$

For 1 min

$K_c = 8$

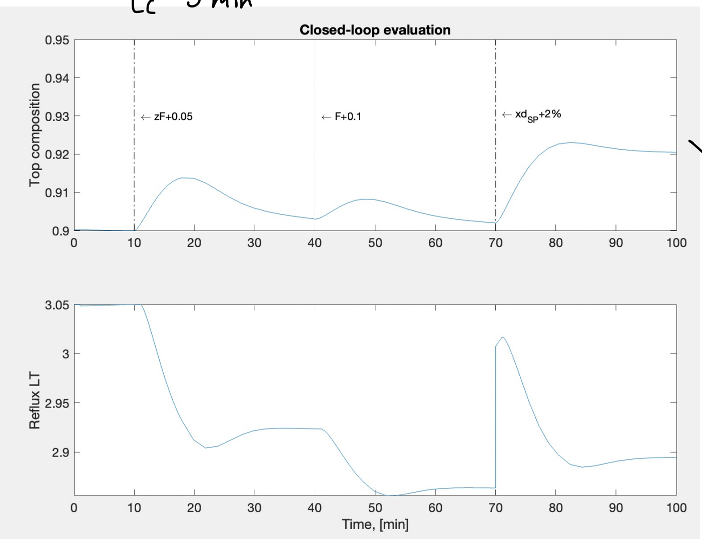
$\tau_I = 14{,}4$

For 0,5 min

$K_c = 6$

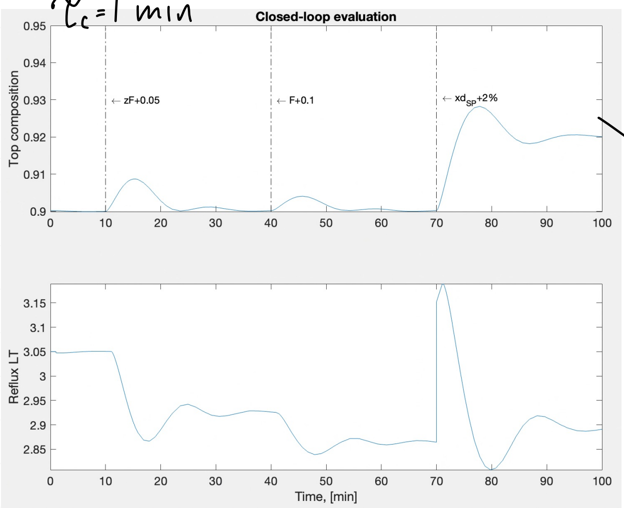
$\tau_I = 19{,}2$

$\tau_c = 3$ min



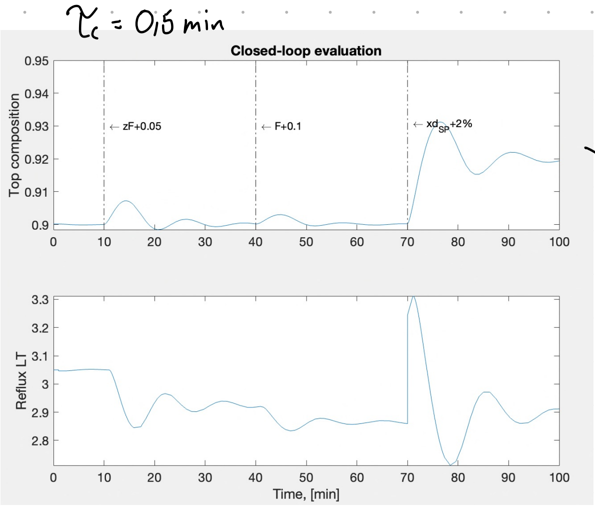
Quite slow, but stable

$\tau_c = 1$ min



Fast response, and quite stable

$\tau_c = 0{,}5$ min

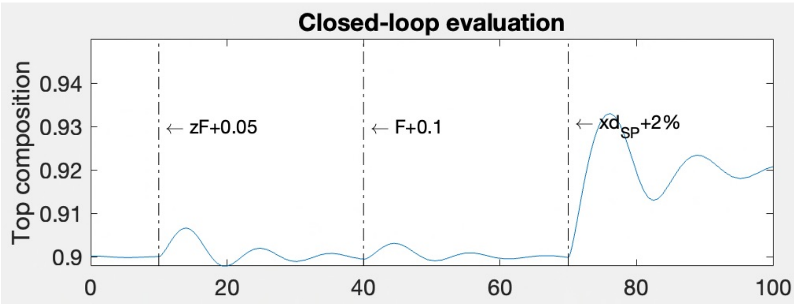


Fast response, unstable

Please comment on the simulations. Which value of τ_c would you recommend? For which τ_c does it become oscillatory?

- I would recommend $\tau_c = 1$ min

- By trial and error, at $\tau_c = 0{,}3$ min, the graph becomes quite oscillatory.



For τ_c= 1 min plot BY HAND the closed-loop response of x_D, x_B and L_t to a 20% step increase in feed rate. Can it be correct that the reflux L_t is reduced?