Exercise 1

Problem 1: (Tabulated properties)

Your team is designing a process that has six streams containing liquid water or steam at different temperatures and pressures, and you need to know the specific internal energy for each. Your teammate has given you the information in Table 1 below, but you need to fill in the missing data. Using the steam tables given as attachment on Blackboard for Exercise Week 1 (using T = 0.01°C, P = 611.7 Pa) as a reference state), determine the values that should go in the two empty cells.

} I'm interpreting this as "all values in the tables have the same reference value".

Table 1: Specific internal energy of liquid water at various conditions

Stream	Temperature (°C)	Pressure (bar)	Phase	$\hat{U}$ (kJ/kg)
1	25	1.0	Liquid	0 (reference state)
2	50	1.0	Liquid	104.5
3	75	1.0	Liquid	209.2
4	100	1.0	Vapor	2401.4
5	100	5.0	Liquid	?
6	200	5.0	Vapor	?

The stream tables uses a diffeent reference state => Need to "shift" the data to our reference state.

Using table A-3, for stream 4

	P = 1 bar				
	T (°C)	$\hat{V}$ (m³/kg)	$\hat{U}$ (kJ/kg)	$\hat{H}$ (kJ/kg)	$\hat{S}$ (kJ/kg·K)
	99.606	1.694	2505.6	2674.9	7.3588
	50				
4	100	1.696	$\boxed{2506.2}$	2675.8	7.3610
	150	1.937	2582.9	2776.6	7.6148
	200	2.172	2658.2	2875.5	7.8356
	250	2.406	2733.9	2974.5	8.0346
	300	2.639	2810.6	3074.5	8.2172
	350	2.871	2888.7	3175.8	8.3866
	400	3.103	2968.3	3278.6	8.5452
	450	3.334	3049.4	3382.8	8.6946
	500	3.566	3132.2	3488.7	8.8361
	550	3.797	3216.6	3596.3	8.9709
	600	4.028	3302.8	3705.6	9.0998
	650	4.259	3390.7	3816.6	9.2234
	700	4.490	3480.4	3929.4	9.3424
	750	4.721	3571.8	4043.9	9.4572
	800	4.952	3665.0	4160.2	9.5681
	850	5.183	3760.0	4278.2	9.6757
	900	5.414	3856.6	4398.0	9.7800
	950	5.645	3955.0	4519.5	9.8813
	1000	5.875	4055.0	4642.6	9.9800

for stream 6

	P = 5 bar				
	T (°C)	$\hat{V}$ (m³/kg)	$\hat{U}$ (kJ/kg)	$\hat{H}$ (kJ/kg)	$\hat{S}$ (kJ/kg·K)
	151.831	0.375	2560.7	2748.1	6.8207
	50				
	100				
	150				
6	200	0.425	$\boxed{2643.3}$	2855.8	7.0610
	250	0.474	2723.8	2961.0	7.2724
	300	0.523	2803.2	3064.6	7.4614
	350	0.570	2883.0	3168.1	7.6346
	400	0.617	2963.7	3272.4	7.7955
	450	0.664	3045.6	3377.7	7.9465
	500	0.711	3129.0	3484.5	8.0892
	550	0.758	3213.9	3592.7	8.2249
	600	0.804	3300.4	3702.5	8.3543
	650	0.851	3388.6	3813.9	8.4784
	700	0.897	3478.5	3927.0	8.5977
	750	0.943	3570.2	4041.8	8.7128
	800	0.990	3663.6	4158.4	8.8240
	850	1.036	3758.6	4276.6	8.9317
	900	1.082	3855.4	4396.6	9.0362
	950	1.129	3953.9	4518.2	9.1377
	1000	1.175	4054.0	4641.4	9.2364

For stream 5, table A-4

	P = 5 bar				
	T (°C)	$\hat{V}$ (m³/kg)	$\hat{U}$ (kJ/kg)	$\hat{H}$ (kJ/kg)	$\hat{S}$ (kJ/kg·K)
	151.831	0.001093	639.5	640.1	1.8604
	0	0.001000	0.0	0.5	−0.0001
	20	0.001002	83.9	84.4	0.2964
	40	0.001008	167.5	168.0	0.5722
	60	0.001017	251.1	251.6	0.8310
	80	0.001029	334.9	335.4	1.0753
5	100	0.001043	$\boxed{418.9}$	419.5	1.3069
	120	0.001060	503.5	504.0	1.5276
	140	0.001080	588.7	589.3	1.7391

The differences between the states should be equal for both reference states.

$$U_5 - U_4 = U_5^{table} - U_4^{table}$$

$$U_5 = U_5^{table} - U_4^{table} + U_4$$

$$U_5 = (418{,}9 - 2506{,}2 + 2401{,}4)\ kJ/kg$$

$$\underline{\underline{U_5 = 314{,}1\ kJ/kg}}$$

Similarily: $U_6 - U_4 = U_6^{table} - U_4^{table}$

$$U_6 = U_6^{table} - U_4^{table} + U_4$$

$$U_6 = (2463{,}3 - 2506{,}2 + 2401{,}4)\ kJ/kg$$

$$\underline{\underline{U_6 = 2358{,}5\ kJ/kg}}$$

Problem 2: (Work and expansion)

100 kg of steam is enclosed in a piston-cylinder device, initially at 300°C and 5 bar. It expands and cools to 200°C and 1 bar. If the external pressure is constant at 1 bar, how much work was done by the steam on the surroundings? (Use the Steam Tables given in Problem 1).

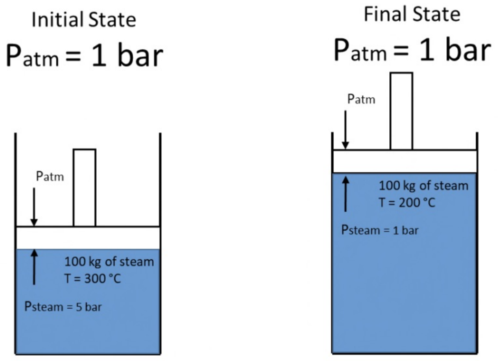

Figure 1: Schematic of piston-cylinder for Problem 2

Slår opp tilstandene i tabell A-3:

Final state

P = 1 bar				
T (°C)	$\hat{V}$ (m³/kg)	$\hat{U}$ (kJ/kg)	$\hat{H}$ (kJ/kg)	$\hat{S}$ (kJ/kg·K)
99.606	1.694	2505.6	2674.9	7.3588
50				
100	1.696	2506.2	2675.8	7.3610
150	1.937	2582.9	2776.6	7.6148
200	$\boxed{2.172}$	2658.2	2875.5	7.8356
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950	5.645	3955.0	4519.5	9.8813
1000	5.875	4055.0	4642.6	9.9800

Initial state

P = 5 bar				
T (°C)	$\hat{V}$ (m³/kg)	$\hat{U}$ (kJ/kg)	$\hat{H}$ (kJ/kg)	$\hat{S}$ (kJ/kg·K)
151.831	0.375	2560.7	2748.1	6.8207
50				
100				
150				
200	0.425	2643.3	2855.8	7.0610
250	0.474	2723.8	2961.0	7.2724
300	$\boxed{0.523}$	2803.2	3064.6	7.4614
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950	1.129	3953.9	4518.2	9.1377
1000	1.175	4054.0	4641.4	9.2364

Expansion of gas, external pressure $\Rightarrow W_E = m\int_{V_1}^{V_2} P_{ext} \cdot dV$

$$= m(V_2 - V_1)P_{ext}$$

$$= 100\text{ kg}(2{,}172 - 0{,}523)\text{ m}^3/\text{kg} \cdot 1\text{ bar}$$

$$= 164{,}9\text{ m}^3 \cdot \text{bar}$$

$$= 164{,}9 \cdot 10^5\text{ m}^3 \cdot \text{Pa}$$

$$\underline{\underline{= 16490\text{ kJ}}}$$

Problem 3 (Internal Energy, Enthalpy ...)

Calculate ΔU and ΔH for 1 kg of water when it is vaporized at the constant temperature of 100°C and the constant pressure of 101.33 kPa. The specific volume of liquid and vapor water at these conditions are 0.00104 and 1.673 m³/kg. For this change, heat in the amount of 2256.9 kJ is added to the water.

$$\Delta U = Q + W_{EC}$$

$$= Q + \left(-m\int_{V_1}^{V_2} P\,dV\right)$$

P=const

$$= Q - mP\int_{V_1}^{V_2} dV$$

$$= Q - mP(V_2 - V_1)$$

$$= 2256{,}9\text{ kJ} - 1\text{ kg} \cdot 101{,}33\text{ k}\overset{J}{\cancel{\text{Pa}}}(1{,}673 - 0{,}00104)\cancel{\text{m}^3}/\cancel{\text{kg}}$$

$$\Delta U \underline{\underline{= 2087{,}5\text{ kJ}}}$$

$$H = U + PV \Rightarrow \Delta H = \Delta U + m\Delta(PV)$$

$$= Q + W_{EC} + mP\Delta V$$

$$= Q - mP\Delta V + mP\Delta V$$

$$= Q$$

$$\Rightarrow \underline{\underline{\Delta H = Q = 2256{,}9\text{ kJ}}}$$

Problem 4 (Enthalpy)

Air at 1 bar and 298.15 K is compressed to 5 bar and 298.15 K by two different mechanically reversible processes:

(a) Cooling at constant pressure followed by heating at constant volume.

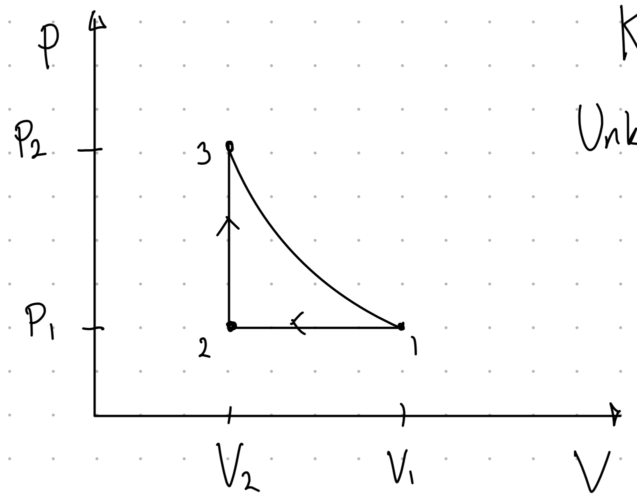
(b) Heating at constant volume followed by cooling at constant pressure.

Calculate the heat and work requirements and ΔU and ΔH of the air for each path (a) and (b). The following heat capacities for air may be assumed independent of temperature:

$C_V = 20.78$ and $C_P = 29.10\ J\ mol^{-1}K^{-1}$

Assume also for air that PV/T is a constant, regardless of the changes it undergoes. At 298.15 K and 1 bar the molar volume of air is 0.02479 $m^3\ mol^{-1}$.

a)



Known: V_1, p_1, T_1, p_3, T_3

Unknown: V_2, p_2, T_2, V_3

From problem:

$P_1 = P_2$

$V_2 = V_3$

Must find V_3, T_2

$$\frac{PV}{T} = \text{const} \Rightarrow \frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3} \Rightarrow V_3 = \frac{P_1}{P_3}\cdot\frac{T_3}{T_1}\cdot V_1 = \frac{1}{5}\cdot\frac{\cancel{298.15K}}{\cancel{298.15K}}\cdot 0{,}02479\ m^3/mol$$

$$\underline{V_3 = V_2 = 4{,}958\cdot 10^{-3}\ m^3/mol}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{P_1V_3}{T_2} \Rightarrow \frac{\cancel{P_1}V_1}{T_1} = \frac{\cancel{P_1}V_3}{T_2}$$

$$\Rightarrow T_2 = \frac{V_3}{V_1}\cdot T_1 = \frac{T_1}{5} = \underline{59{,}63\ K}$$

$$W_{12} = -\int_{V_1}^{V_2} P\,dV \underset{\substack{\uparrow\\ P=\text{constant}}}{=} -P(V_2 - V_1) = -10^5\,Pa\,(4{,}958\cdot 10^{-3}\ m^3/mol - 0{,}02479\ m^3/mol)$$

$$W_{12} = 1983{,}2\ J/mol$$

$$W_{13} = -\int_{V_1}^{V_2} P\,dV = -\int_{V_1}^{V_2} \frac{nRT}{V}\,dV = -nRT\ln\left(\frac{V_2}{V_1}\right) = 0$$

$V_1 = V_2 \Rightarrow \ln(\dots) = 0$

$$\underline{\underline{W = 1983{,}2\ J/mol}}$$

$$Q_{12} = C_p \Delta T = 29{,}10 \ \mathrm{J/mol \cdot K} \left(59{,}63 - 298{,}15\right) K = -6940{,}9 \ \mathrm{J/mol}$$

$$Q_{23} = C_V \Delta T = 20{,}78 \ \mathrm{J/mol \cdot K} \left(298{,}15 - 59{,}63\right) = 4956{,}4 \ \mathrm{J/mol}$$

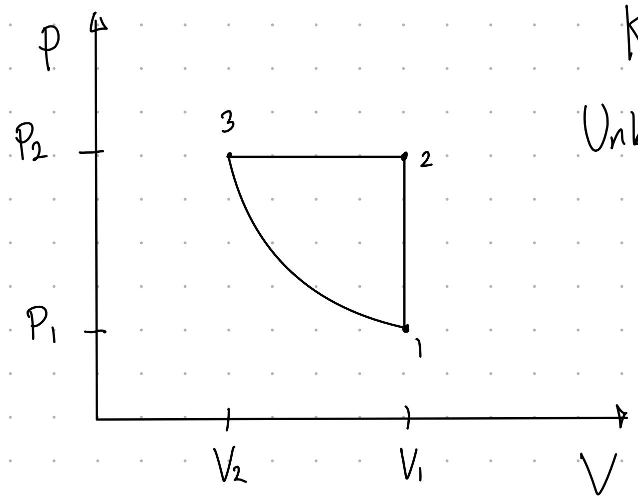
$$\underline{\underline{Q = Q_{12} + Q_{23} = -1984{,}5 \ \mathrm{J/mol}}}$$

$$\Delta U = Q + W_{E/C} = 0$$ (I'm assuming a rounding error in my answers)

$$\Delta H = \Delta U + \underbrace{W_S}_{=0} = \Delta U = 0$$

$$\underline{\underline{\Delta H = \Delta U = 0}}$$

b)



Known: V_1, p_1, T_1, p_3, T_3

Unknown: V_2, p_2, T_2, V_3

From problem:

$V_1 = V_2$

$p_2 = p_3$

Must find V_3, T_2

$$\frac{PV}{T} = \text{const} \Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \Rightarrow V_3 = \frac{P_1}{P_3} \cdot \frac{T_3}{T_1} \cdot V_1 = \frac{1}{5} \cdot 0{,}02479 \ \mathrm{m^3/mol} = 4{,}958 \cdot 10^{-3} \ \mathrm{m^3/mol}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_1}{T_2} \Rightarrow \frac{P_1 \cancel{V_1}}{T_1} = \frac{P_3 \cancel{V_1}}{T_2} \Rightarrow T_2 = T_1 \cdot \frac{P_3}{P_1} = 298{,}15 \cdot \frac{5}{1} K = 1490{,}75 K$$

Similerily to a), $W_{12} = 0$ as the volume doesn't change.

$$W = W_{23} = -\int_{V_1}^{V_2} P \, dV \underset{P = \text{constant}}{=} -P\left(V_2 - V_1\right) = -5 \cdot 10^5 \ \mathrm{Pa} \left(4{,}958 \cdot 10^{-3} \ \mathrm{m^3/mol} - 0{,}02479 \ \mathrm{m^3}\right)$$

$$\Rightarrow \underline{\underline{W = 9916 \ \mathrm{J/mol}}}$$

$$Q_{12} = C_v \Delta T = 20{,}78 \cdot (1490{,}75 - 298{,}15) = 24782 \text{ J/mol}$$

$$Q_{23} = C_p \Delta T = 29{,}10 \text{ J/mol} (298{,}15 - 1490{,}75) = -34705 \text{ J/mol}$$

$$\underline{\underline{Q = Q_{12} + Q_{23} = -9922{,}4 \text{ J/mol}}}$$

$$\Delta U = Q + W_{E/C} = (9916 - 9922{,}4) \text{ J/mol} = -6{,}4 \text{ J/mol} \approx 0 \text{ J/mol}$$

$$\Delta U = 0$$

(assume rounding error)

$$\Delta H = \Delta U + \Delta(PV) = 0 + P_1V_1 - P_3V_3 = 1 \text{ bar} \cdot V_1 - 5 \text{ bar} \cdot \tfrac{1}{5} V_1 = 0$$

$$\underline{\underline{\Delta U = \Delta H = 0}}$$

Problem 5 : Mass and energy balances application (The complete energy balance)

An ideal gas has C_P= (7/2)R. One mole of this gas is confined in a piston-cylinder device. Initially, the gas is at T = 300 K and P = 1 bar (Figure 2). If the gas is compressed isothermally to P = 7 bar, find the amounts of work and heat associated with the process. List the assumptions taken to simplify the energy balance for calculation.

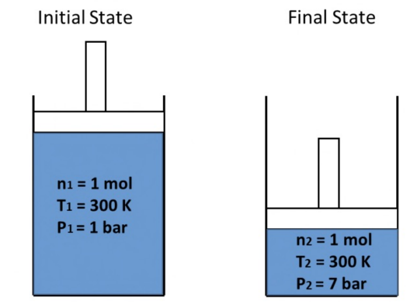


Figure 2: Schematic of piston-cylinder for Problem 5

Energy balance for closed system:

- Assume no mass transfer across the bounderies.
- Assume Ek, Ep and Ws to be very small compared to rest of the terms
- Assume ideal gas
- Assume constant Cp

We get: $\Delta U = Q + W_C$, $\Delta U = 0$, because the compression is isotherm

$$\Rightarrow Q = -W_C = -\left(-\int_{V_1}^{V_2} P\,dV\right) \underset{\text{ideal gas}}{=} nRT \int_{V_1}^{V_2} \frac{1}{V}\,dV = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$\underset{\text{ideal gas law again}}{=} nRT \ln\left(\frac{P_1}{P_2}\right) = 1 \cancel{\text{mol}} \cdot 8{,}314 \text{ J}/\cancel{\text{K}}\cdot\cancel{\text{mol}} \cdot 300 \cancel{\text{K}} \ln\left(\frac{1}{7}\right) = -4853{,}5 \text{ J}$$

$$\Rightarrow \underline{\underline{Q = -4853{,}5 \text{ J}, \; W = 4853{,}5 \text{ J}}}$$