Problem 3: Rate of drying

Test drying of a food product was carried out in an insulated tray drier. The temperature of the drying air was 60°C and the dew point 20°C. The dry weight of the material was 3.765kg and the surface area was 0.186m^2. The test data are given in the table below.

Weight of dried material at different times during the drying test												
Time t [h]	0,0	0,4	0,8	1,4	2,2	3,0	4,2	5,0	7,0	9,0	12,0	25,0
Weight of dried material m_dried [kg]	4,944	4,885	4,808	4,699	4,554	4,404	4,241	4,15	4,019	3,978	3,955	3,955

a) Determine the equilibrium moisture content.
   Total moisture content is:

$$X_{total} = \frac{m_{wet\ material} - m_{dry\ solid}}{m_{dry\ solid}} \left[\frac{kg_{\ H2O}}{kg_{\ dry\ solid}}\right]$$

   Equilibrium moisture content is reached at the end of the drying test when no further decrease in moisture content is achieved between the 12th and 25th hour of drying.

$$X^* = \frac{m_{wet\ material_{end\ of\ test}} - m_{dry\ solid}}{m_{dry\ solid}} = \frac{3.955 - 3.765}{3.765} = \mathbf{0.0505} \left[\frac{kg_{\ H2O}}{kg_{\ dry\ solid}}\right]$$

b) Determine the drying rate $R\,[kg_{\ water}/m^2.\,s]$ as function of the free moisture content.

   Free moisture content is defined as:

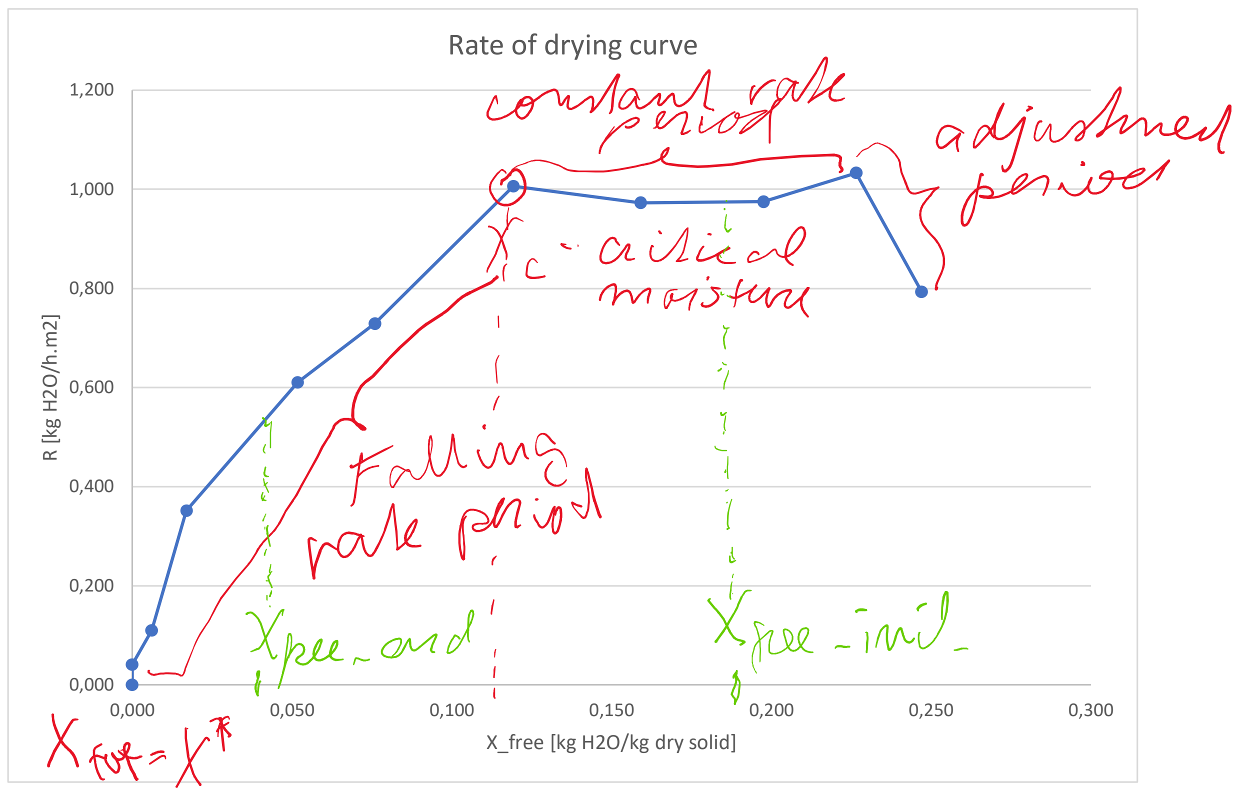
$$X_{free} = X_{total} - X^*$$

   Rate of drying is defined as follows:

$$R = -\frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \frac{dX}{dt} \quad \left[\frac{kg_{\ H2O}}{s\,.\,m^2}\right]$$

Weight of dried material at different times during the drying test												
Time t [h]	0,0	0,4	0,8	1,4	2,2	3,0	4,2	5,0	7,0	9,0	12,0	25,0
Weight of dried material m_dried [kg]	4,944	4,885	4,808	4,699	4,554	4,404	4,241	4,15	4,019	3,978	3,955	3,955
Moisture content **X_total** [kg H2O/kg dry solid]	0,3131	0,2975	0,2770	0,2481	0,2096	0,1697	0,1264	0,1023	0,0675	0,0566	**0,0505**	**0,0505**
Free moisture **X_free** [kg H2O/kg dry solid]	0,263	0,247	0,227	0,198	0,159	0,119	0,076	0,052	0,017	0,006	0,000	0,000
DX/Dt [(kg H2O/kg dry solid)/h]		-0,0392	-0,0511	-0,0483	-0,0481	-0,0498	-0,0361	-0,0302	-0,0174	-0,0054	-0,0020	0,0000
R = -m_dry/A_dried.DX/Dt [kg H2O/h . m2]		0,793	1,033	0,975	0,973	1,006	0,729	0,610	0,351	0,110	0,041	0,000

c) Draw the rate of drying curve. Identify the different drying periods in the figure.



d) Calculate the time needed for drying the food product under same conditions as the drying test conditions from total water content of 0.25 [kg_water/kg_ dry_material] to final total water content of 0.10 [kg_water/kg_ dry_material].

Initial moisture content is:

$$X_{tot\_init} = 0.25 \quad \left[\frac{kg\ H2O}{kg\ _{dry\ solid}}\right]$$

$$X_{free\_init} = X_{tot\_init} - X^* = 0{,}25 - 0{,}0505 = 0{,}1995 \quad \left[\frac{kg\ H2O}{kg\ _{dry\ solid}}\right]$$

Final moisture content is:

$$X_{tot\_end} = 0.10 \quad \left[\frac{kg\ H2O}{kg\ _{dry\ solid}}\right]$$

$$X_{free\_end} = X_{tot\_end} - X^* = 0{,}10 - 0{,}0505 = 0{,}0495 \quad \left[\frac{kg\ H2O}{kg\ _{dry\ solid}}\right]$$

The critical moisture content is: $X_{crit} = 0.119 \quad \left[\frac{kg\ H2O}{kg\ _{dry\ solid}}\right]$

The critical rate of drying is:

$$R_c = 1.006 \quad \left[\frac{kg_{H2O}}{s\ .\ m^2}\right]$$

Drying from the specified initial moisture content to the specified final moisture content covers both constant rate of drying period [X_init; X_critical] and falling rate of drying period [X_critical; X_end].

Time of drying in constant rate of drying period [X_init; X_critical]:

Definition of rate of drying:

$$R = -\frac{m_{dry\,solid}}{A_{dried\,solid}} . \frac{dX}{dt} \quad \left[\frac{kg_{H2O}}{s\,.\,m^2}\right]$$

Integration over the course of constant rate of drying (X_free between [0,1995; 0,119]) gives:

$$\int_{t_1=0}^{t_2=t_c} dt = -\frac{m_{dry\,solid}}{A_{dried\,solid}} . \int_{X_1}^{X_c} \frac{dX}{R_c}$$

$$t_c = \frac{m_{dry\,solid}}{A_{dried\,solid} . R_c} . (X_1 - X_c) = \frac{3.765}{0{,}186\,.\,1{,}006}(0.1995 - 0.119) = \mathbf{1.63\ h}$$

Integration over the course of falling rate of drying (X_free between [0,119; 0,0495]) assuming linear function from the point [R_c; X_c] through the origin gives:

$$\int_{t_1=t_c}^{t_2=t_{end}} dt = -\frac{m_{dry\,solid}}{A_{dried\,solid}} . \int_{X_c}^{X_{end}} \frac{dX}{R}$$

For the linear decrease in the rate of drying described by $R = aX$ it could be derived:

$dR = a.dX$ and $a = \frac{R_c}{X_c}$ and $\frac{R_c}{R_{end}} = \frac{X_c}{X_{end}}$

Then:

$$t_{c\_end} = \frac{m_{dry\,solid}}{A_{dried\,solid}} . \frac{X_c}{R_c}\, ln \frac{R_c}{R_{end}} = \frac{m_{dry\,solid}}{A_{dried\,solid}} . \frac{X_c}{R_c}\, ln \frac{X_c}{X_{end}}$$

$$t_{c\_end} = \frac{3.765}{0.186} . \frac{0.119}{1{,}006}\, ln \frac{0.119}{0.0495} = \mathbf{2.14\ h}$$

Total time needed for the specified drying process is:

$$t = t_c + t_{c\_end} = 1.63 + 2.14 = \mathbf{3.77\ h}$$