Problem 6: Batch evaporation crystallizer

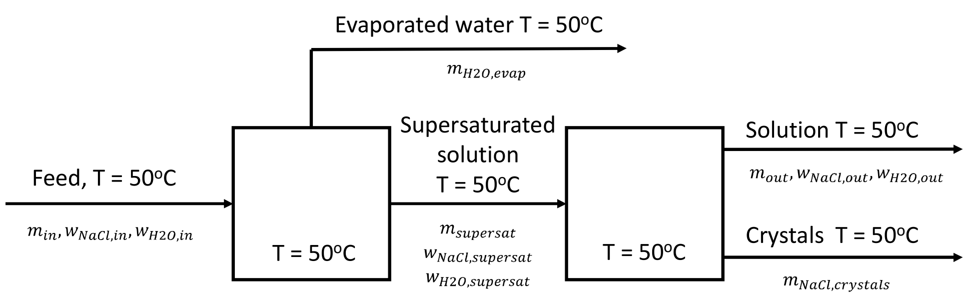
A stirred batch evaporation crystallizer for sodium chloride is operated at 50°C. The crystallizer is fed with sodium chloride solution saturated at the operation temperature. Total mass of 2432kg water is evaporated so that the mass of crystals in the supersaturated solution becomes 473kg per m^3 supersaturated solution.

Following data are provided:

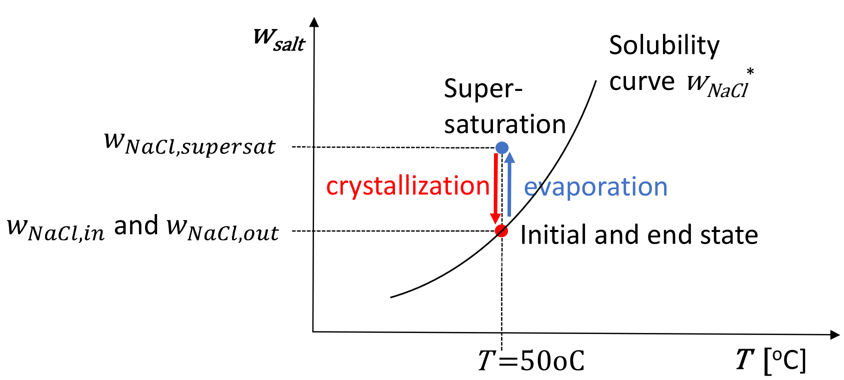
Solubility of NaCl at 50°C	37 [kg/100kg water]
Density of saturated solution at 50°C	1200 [kg/m^3]
Density of supersaturated solution at 50°C (ass. $\Delta V \approx -2m^3$)	1192 [kg/m^3]
Crystal seeds size	10^{-4} [m]
Product crystal size	10^{-3} [m]
Crystal growth rate	8.10^{-8} [m/s]

a) Draw a commented sketch of the process and an illustration of the process in concentration – temperature diagram.

Sketch of the process:



Concentration-temperature diagram:



b) Calculate the total volume of the solution fed into the crystallizer.

The overall mass balance is:

$$m_{in} = m_{supersat} + m_{H2O,evap} = m_{out} + m_{NaCl,crystals} + m_{H2O,evap}$$

The mass balance for water is:

$$m_{in}.w_{H2O,in} = m_{supersat}.w_{H2O,supersat} + m_{H2O,evap} = m_{out}.w_{H2O,out} + m_{H2O,evap}$$

The mass balance for the salt is:

$$m_{in}.w_{NaCl,in} = m_{supersat}.w_{NaCl,supersat} = m_{out}.w_{NaCl,out} + m_{NaCl,crystals}$$

The mass fractions of salt and water in the feed solution and in the mother solution are given by the solubility of NaCl at 50°C given as 37 [kg/100kg water] since both these solutions are saturated solutions.

$$w_{NaCl} = \frac{m_{NaCl}}{m_{H2O} + m_{NaCl}} = \frac{37}{100 + 37} = 0.27$$

$$w_{H2O} = 1 - w_{NaCl} = 1 - 0.27 = 0.73$$

The mass fractions in the supersaturated solutions are:

$$w_{NaCl,supersat} = \frac{\frac{m_{NaCl}}{V_{supersat}}}{\frac{m_{supersat}}{V_{supersat}}} = \frac{\frac{m_{NaCl}}{V_{supersat}}}{\rho_{supersat}} = \frac{473}{1192} = 0.3968$$

$$w_{H2O,supersat} = 1 - w_{NaCl,supersat} = 1 - 0.3968 = 0.6032$$

By combining 2 balance equations:
Expressing $m_{supersat}$ from the overall balance

$$m_{in} - m_{H2O,evap} = m_{supersat}$$

Inserting for $m_{supersat}$ into the salt balance

$$m_{in}.w_{NaCl,in} = (m_{in} - m_{H2O,evap}).w_{NaCl,supersat}$$

Provides equation for m_{in}

$$m_{in}.(w_{NaCl,supersat} - w_{NaCl,in}) = m_{H2O,evap}.w_{NaCl,supersat}$$

$$m_{in} = (m_{H2O,evap}.w_{NaCl,supersat})/(w_{NaCl,supersat} - w_{NaCl,in})$$

$$m_{in} = \frac{(2432 . 0.3968)}{(0.3968 - 0.27)} = 7611\text{kg}$$

The total volume of the solution is then

$$V_{in} = \frac{m_{in}}{\rho_{sol}} = \frac{7611}{1200} = \mathbf{6.34\ m^3}$$

c) Calculate the crystallization yield.
By combining 2 balance equations
From the overall mass balance calculate $m_{supersat}$ and express m_{out}:

$$m_{supersat} = m_{in} - m_{H2O,evap} = 7611 - 2432 = 5179\text{kg}$$

$$m_{out} = m_{supersat} - m_{NaCl,crystals}$$

Insert m_{out} into the salt mass balance:

$$m_{supersat}.w_{NaCl,supersat} = m_{out}.w_{NaCl,out} + m_{NaCl,crystals}$$

$$m_{supersat}.w_{NaCl,supersat} = \left(m_{supersat} - m_{NaCl,crystals}\right).w_{NaCl,out} + m_{NaCl,crystals}$$

$$m_{supersat}\left(w_{NaCl,supersat} - w_{NaCl,out}\right) = m_{NaCl,crystals}.\left(1 - w_{NaCl,out}\right)$$

That provides for the mass of produced crystals:

$$m_{NaCl,crystals} = m_{supersat}\frac{\left(w_{NaCl,supersat} - w_{NaCl,out}\right)}{\left(1 - w_{NaCl,out}\right)}$$

$$m_{NaCl,crystals} = 5179.\frac{0.3968 - 0.27}{1 - 0.27} = 899.6\text{kg}$$

Then the crystallization yield is given as:

$$Yield = \frac{m_{NaCl,crystals}}{m_{NaCl,in}}.100 = \frac{m_{NaCl,crystals}}{m_{in}.w_{NaCl,in}}.100 = \frac{899{,}6}{7611.0.27}.100 = \mathbf{44}\%$$

d) Calculate the crystallization time needed per batch assuming validity of the McCabe law of crystal growth $\Delta L = G.\Delta\tau$
From the McCabe law of crystal growth: $\Delta\tau = \frac{\Delta L}{G}$

$$(\tau - \tau_0) = \frac{(L - L_0)}{G} = \frac{10^{-3} - 10^{-4}}{8.10^{-8}} = 11250\text{s} = \mathbf{3.13h}$$

McCabe law of crystal growth

- Growth uniform in all directions
- Growth rate independent of size

$$G = \frac{\Delta L}{\Delta\tau} = K\Delta c_A$$

- For constant super-saturation

$$\boxed{\Delta L = G.\Delta\tau}$$