

Problem 5: Rate of drying

Test drying of a food product was carried out in an insulated tray drier. The temperature of the drying air was 60°C and the dew point 20°C. The dry weight of the material was 3.765kg and the surface area was 0.186m². The test data are given in the table below.

Weight of dried material at different times during the drying test												
Time t [h]	0,0	0,4	0,8	1,4	2,2	3,0	4,2	5,0	7,0	9,0	12,0	25,0
Weight of dried material m_dried [kg]	4,944	4,885	4,808	4,699	4,554	4,404	4,241	4,15	4,019	3,978	3,955	3,955

a) Determine the equilibrium moisture content.

Total moisture content is:

$$X_{total} = \frac{m_{wet\ material} - m_{dry\ solid}}{m_{dry\ solid}} \left[\frac{kg\ H_2O}{kg\ dry\ solid} \right]$$

Equilibrium moisture content is reached at the end of the drying test when no further decrease in moisture content is achieved between the 12th and 25th hour of drying.

$$X^* = \frac{m_{wet\ material\ end\ of\ test} - m_{dry\ solid}}{m_{dry\ solid}} = \frac{3.955 - 3.765}{3.765} = 0.0505 \left[\frac{kg\ H_2O}{kg\ dry\ solid} \right]$$

b) Determine the drying rate $R [kg\ water/m^2 \cdot s]$ as function of the free moisture content.

Free moisture content is defined as:

$$X_{free} = X_{total} - X^*$$

Rate of drying is defined as follows:

$$R = - \frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \frac{dX}{dt} \left[\frac{kg\ H_2O}{s \cdot m^2} \right]$$

Weight of dried material at different times during the drying test												
Time t [h]	0,0	0,4	0,8	1,4	2,2	3,0	4,2	5,0	7,0	9,0	12,0	25,0
Weight of dried material m_dried [kg]	4,944	4,885	4,808	4,699	4,554	4,404	4,241	4,15	4,019	3,978	3,955	3,955
Moisture content X_total [kg H2O/kg dry solid]	0,3131	0,2975	0,2770	0,2481	0,2096	0,1697	0,1264	0,1023	0,0675	0,0566	0,0505	0,0505
Free moisture X_free [kg H2O/kg dry solid]	0,263	0,247	0,227	0,198	0,159	0,119	0,076	0,052	0,017	0,006	0,000	0,000
DX/Dt [(kg H2O/kg dry solid)/h]		-0,0392	-0,0511	-0,0483	-0,0481	-0,0498	-0,0361	-0,0302	-0,0174	-0,0054	-0,0020	0,0000
R = -m_dry/A_dried.DX/Dt [kg H2O/h . m2]		0,793	1,033	0,975	0,973	1,006	0,729	0,610	0,351	0,110	0,041	0,000

c) Draw the rate of drying curve. Identify the different drying periods in the figure.



- d) Calculate the time needed for drying the food product under same conditions as the drying test conditions from total water content of 0.25 [kg_{water}/kg_{dry material}] to final total water content of 0.10 [kg_{water}/kg_{dry material}].

Initial moisture content is:

$$X_{\text{tot_init}} = 0.25 \left[\frac{\text{kg H}_2\text{O}}{\text{kg dry solid}} \right]$$

$$X_{\text{free_init}} = X_{\text{tot_init}} - X^* = 0.25 - 0.0505 = 0.1995 \left[\frac{\text{kg H}_2\text{O}}{\text{kg dry solid}} \right]$$

Final moisture content is:

$$X_{\text{tot_end}} = 0.10 \left[\frac{\text{kg H}_2\text{O}}{\text{kg dry solid}} \right]$$

$$X_{\text{free_end}} = X_{\text{tot_end}} - X^* = 0.10 - 0.0505 = 0.0495 \left[\frac{\text{kg H}_2\text{O}}{\text{kg dry solid}} \right]$$

The critical moisture content is:

$$X_{\text{crit}} = 0.119 \left[\frac{\text{kg H}_2\text{O}}{\text{kg dry solid}} \right]$$

The critical rate of drying is:

$$R_c = 1.006 \left[\frac{\text{kg H}_2\text{O}}{\text{s} \cdot \text{m}^2} \right]$$

Drying from the specified initial moisture content to the specified final moisture content covers both constant rate of drying period [X_init; X_critical] and falling rate of drying period [X_critical; X_end].

Time of drying in constant rate of drying period [X_init; X_critical]:

Definition of rate of drying:

$$R = -\frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \frac{dX}{dt} \quad \left[\frac{kg_{H_2O}}{s \cdot m^2} \right]$$

Integration over the course of constant rate of drying (X_free between [0,1995; 0,119]) gives:

$$\int_{t_1=0}^{t_2=t_c} dt = -\frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \int_{X_1}^{X_c} \frac{dX}{R_c}$$

$$t_c = \frac{m_{dry\ solid}}{A_{dried\ solid} \cdot R_c} \cdot (X_1 - X_c) = \frac{3.765}{0.186 \cdot 1.006} (0.1995 - 0.119) = \mathbf{1.63\ h}$$

Integration over the course of falling rate of drying (X_free between [0,119; 0,0495]) assuming linear function from the point [R_c; X_c] through the origin gives:

$$\int_{t_1=t_c}^{t_2=t_{end}} dt = -\frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \int_{X_c}^{X_{end}} \frac{dX}{R}$$

For the linear decrease in the rate of drying described by $R = aX$ it could be derived:

$$dR = a \cdot dX \quad \text{and} \quad a = \frac{R_c}{X_c} \quad \text{and} \quad \frac{R_c}{R_{end}} = \frac{X_c}{X_{end}}$$

Then:

$$t_{c_end} = \frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \frac{X_c}{R_c} \ln \frac{R_c}{R_{end}} = \frac{m_{dry\ solid}}{A_{dried\ solid}} \cdot \frac{X_c}{R_c} \ln \frac{X_c}{X_{end}}$$

$$t_{c_end} = \frac{3.765}{0.186} \cdot \frac{0.119}{1.006} \ln \frac{0.119}{0.0495} = \mathbf{2.14\ h}$$

Total time needed for the specified drying process is:

$$t = t_c + t_{c_end} = 1.63 + 2.14 = \mathbf{3.77\ h}$$