

Proposed solution exam H18

Problem 1: Drying (25%)

a) Percentage relative humidity of the air (H_p) 2p

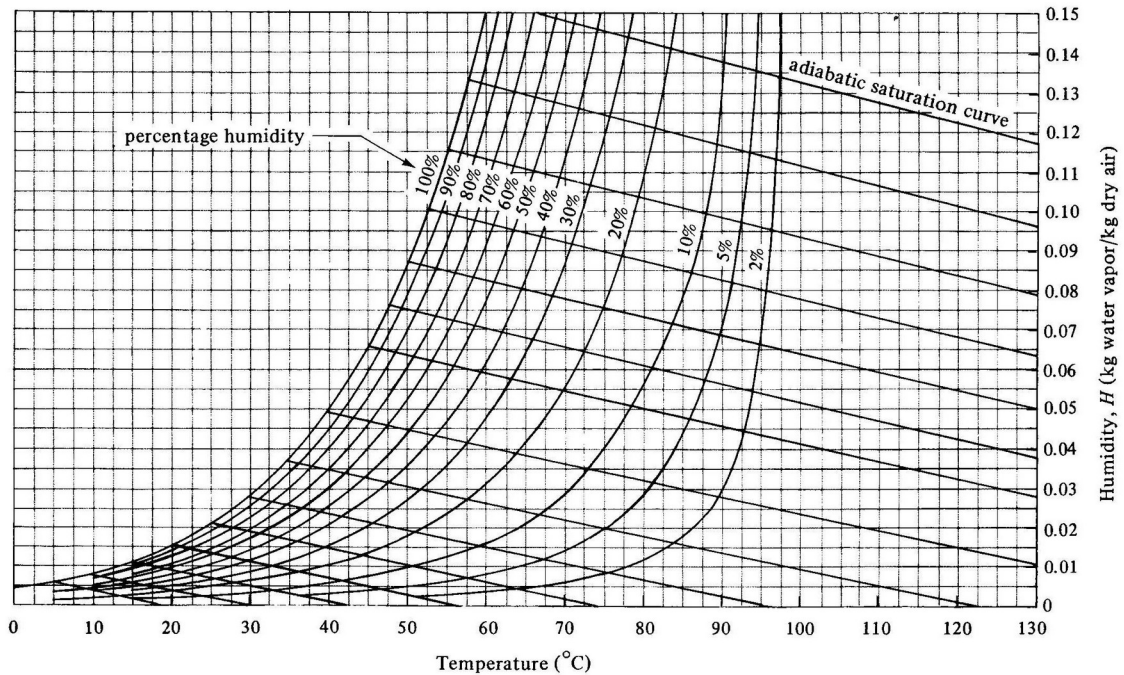
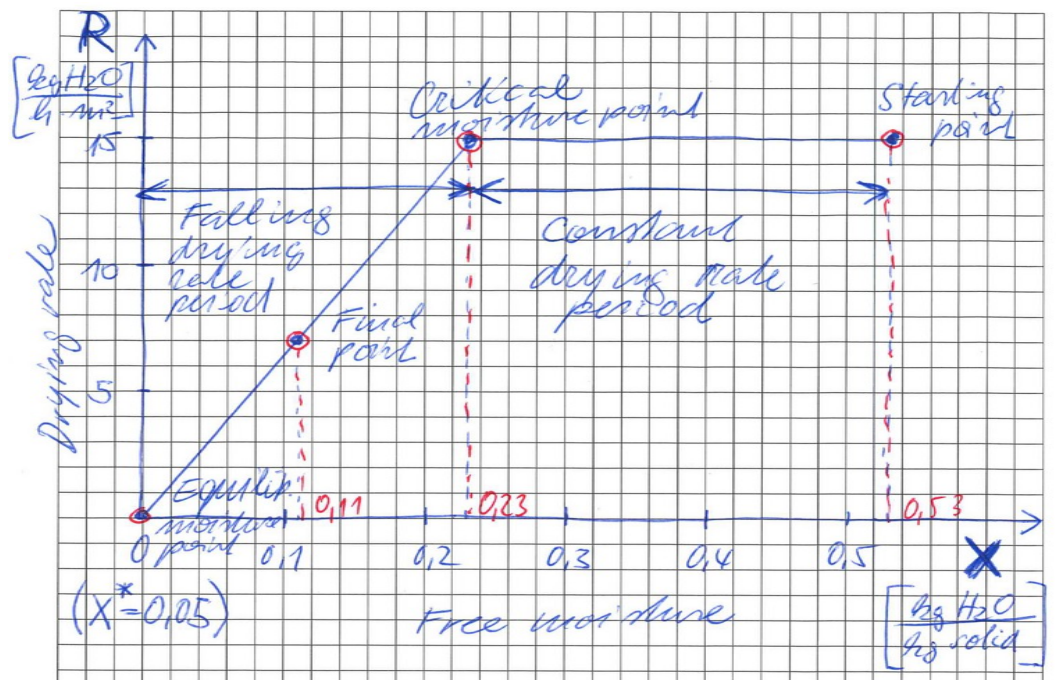


FIGURE 9.3-2. Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)

b) Drying curve based on given information 7p



- c) From the general definition of drying rate 7p

$$R = \frac{-M_{dry\ solid}}{A_{dried\ solid}} \cdot \frac{dX}{dt}$$

$$\int_{t_1=0}^{t_2=t} dt = \frac{-M_s}{A} \cdot \int_{X_1}^{X_2} \frac{dX}{R_c}$$

For constant drying rate

$$t = \frac{M_s}{A \cdot R_c} \cdot (X_1 - X_2) = \frac{100}{1 \cdot 15} (0,53 -$$

- d) For the falling drying rate linear function is assumed 7p

$$R = aX \quad dR = a dX$$

$$t = \frac{-M_s}{A} \cdot \int_{X_c}^{X_2} \frac{dX}{R} = \frac{M_s}{aA} \cdot \int_{R_2}^{R_c} \frac{dR}{R} = \frac{M_s}{aA} \ln \frac{R_c}{R_2}$$

$$R_c = a X_c$$

$$a = \frac{R_c}{X_c}$$

$$R_2 = a X_2$$

$$\frac{R_c}{R_2} = \frac{X_c}{X_2}$$

$$t = \frac{M_s X_c}{A R_c} \ln \frac{R_c}{R_2} = \frac{M_s X_c}{A R_c} \ln \frac{X_c}{X_2} = \frac{100 \cdot 0,23}{1 \cdot 15} \ln$$

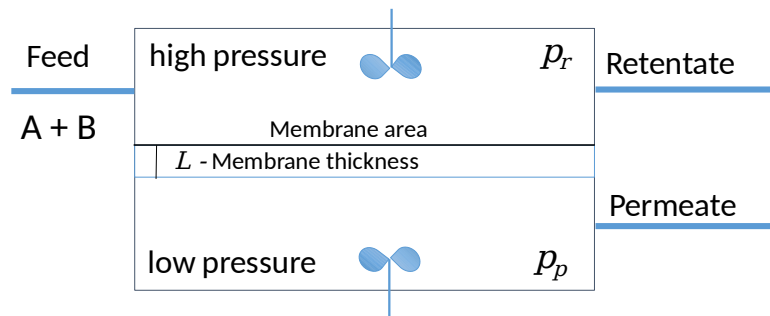
- e) When the point of critical free moisture contents is reached, there is not enough water to form continuous film on the surface. The wetted surface fraction is continuously decreasing through this entire period until the surface is completely dry.

(At that point normally the second falling-rate period starts. The plane of evaporation is no longer at the surface, the heat of evaporation has to be transported through the solid to the evaporation zone and the evaporated water has to move from within the solid to the air flow.)

2p

Problem 2: Membrane gas separation

a) Complete mixing model 3p



The underlying assumptions of a complete mixing model.

- Assumption of complete mixing on both sides of the membranes means no concentration gradient along the membrane - we do not need local differential balance.
- It is further assumed that there is no mass transfer resistance in the gas phases which means no concentration gradient from the bulk liquid towards the membrane surface.
- The only mass transfer resistance is the membrane.

b) The rate transfer equation: 8p

$$y_{Ar} = \frac{y_{Ap} [R(1 - y_{Ap})(\alpha - 1) + 1]}{\alpha - (\alpha - 1)y_{Ap}}$$

can be derived by combining the flux equations and introducing:

$$\alpha = \frac{P_A}{P_B} \text{ membrane selectivity}$$

$$R = \frac{p_p}{p_r} \text{ relative pressure}$$

1) CO₂ capture from flue gas

$$\alpha_1 = \frac{P_{CO_2}}{P_{N_2}} = \frac{6 \cdot 10^{-7}}{3 \cdot 10^{-9}} = 200 \quad R_1 = \frac{p_p}{p_r} = \frac{0,1}{4}$$

2) CO₂ capture from biogas and 3) CO₂ capture from natural gas

$$\alpha_{2,3} = \frac{P_{CO_2}}{P_{CH_4}} = \frac{6 \cdot 10^{-7}}{1,2 \cdot 10^{-8}} = 50 \quad R_2 = \frac{p_p}{p_r} = \frac{1}{7} \quad R_3 = \frac{p_p}{p_r} = \frac{1}{70}$$

The theoretical minimum concentration in the retentate will be achieved if $\theta = 1$, i.e. if

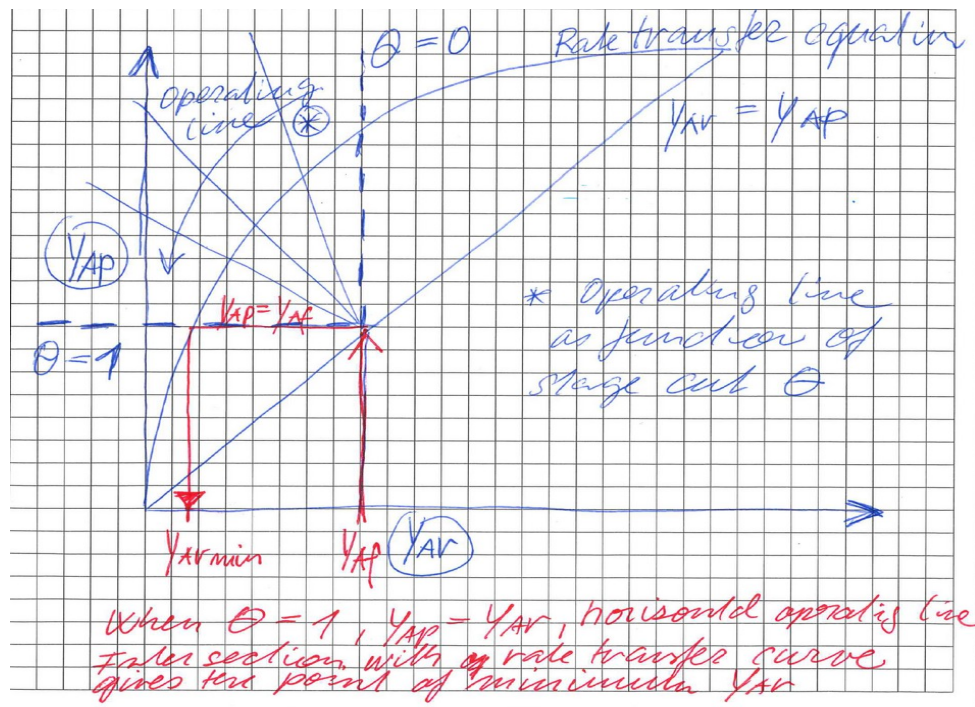
$$y_{Ap} = y_{Af}$$

Then

$$y_{Ar} = \frac{y_{Af} [R(1 - y_{Af})(\alpha - 1) + 1]}{\alpha - (\alpha - 1)y_{Af}}$$

$$\begin{aligned}
 1) \quad y_{CO_2, r, min} &= \frac{0,13 \left[\frac{0,1}{4} (1 - 0,13) (200 - 1) + 1 \right]}{200 - (200 - 1) 0,13} = 3,98 \cdot 10^{-3} \\
 2) \quad y_{CO_2, r, min} &= \frac{0,40 \left[\frac{1}{7} (1 - 0,40) (50 - 1) + 1 \right]}{50 - (50 - 1) 0,40} = 6,84 \cdot 10^{-2} \\
 3) \quad y_{CO_2, r, min} &= \frac{0,10 \left[\frac{1}{70} (1 - 0,10) (50 - 1) + 1 \right]}{50 - (50 - 1) 0,10} = 3,6 \cdot 10^{-3}
 \end{aligned}$$

Graphical illustration of the minimum retentate concentration.



c) Case 1 **5p**

The required volume fraction of component A in the permeate is $y_{Ap} = 0,95$.

It is further given that: $\theta = \frac{V_p}{V_f} = 0,1$

From the given feed flow $V_f = 5 \cdot 10^5 \text{ m}^3 (\text{STP}) / \text{hour}$ and the defined stage cut:

$$V_p = 5 \cdot 10^4 \text{ m}^3 (\text{STP}) / \text{hour}$$

Formulate the overall material balance equation and the balance equation for component A.
(Volumetric flow is given so we use it as basis)

Overall balance:

$$V_f = V_r + V_p$$

Then

$$V_r = V_f - V_p = 4,5 \cdot 10^5 \text{ m}^3 (\text{STP}) / \text{hour}$$

$$\text{Balance for component A: } V_f y_{Af} = V_r y_{Ar} + V_p y_{Ap}$$

Then

$$y_{Ar} = \frac{(V_f y_{Af} - V_p y_{Ap})}{V_r} = \frac{(0,13 \cdot 5 \cdot 10^5 - 0,95 \cdot 5 \cdot 10^4)}{4,5 \cdot 10^5} = 0,04$$

The required capture rate is 85%.

The achieved capture rate is:

$$100 \cdot \frac{(V_f y_{Af} - V_r y_{Ar})}{V_f y_{Af}} = 100 \cdot \frac{(0,13 \cdot 5 \cdot 10^5 - 0,04 \cdot 4,5 \cdot 10^5)}{0,13 \cdot 5 \cdot 10^5} = 72,3 \%$$

This is lower than the required capture rate.

d) Case 3 **5p**

The flux equation is given as: $J_{AV} = \frac{P_A}{L} (p_r y_{Ar} - p_p y_{Ap})$

The required retentate mole fraction is $y_{Ar} = 0,025$

The permeate mole fraction can be calculated from the overall balance and given stage cut:

$$\theta = \frac{V_p}{V_f} = 0,1 \quad \text{then} \quad V_p = \theta \cdot V_f = 0,1 \cdot 10^4 = 10^3 \text{ m}^3 (\text{STP}) / \text{hour}$$

$$\text{and} \quad V_r = V_f - V_p = 10^4 - 0,1 \cdot 10^4 = 0,9 \cdot 10^4 \text{ m}^3 (\text{STP}) / \text{hour}$$

$$y_{Ap} = \frac{(V_f y_{Af} - V_r y_{Ar})}{V_p} = \frac{(10^4 \cdot 0,1 - 0,9 \cdot 10^4 \cdot 0,025)}{10^3} = 0,775$$

$$\text{The flux is: } J_{AV} = \frac{V_p \cdot y_{Ap}}{A} = \frac{\theta \cdot V_f \cdot y_{Ap}}{A}$$

Then the membrane area can be calculate as:

$$A = \frac{\theta \cdot V_f \cdot y_{Ap}}{\frac{P_A}{L} (p_r y_{Ar} - p_p y_{Ap})} = \frac{0,1 \cdot 10^4 \cdot 0,775}{\frac{6 \cdot 10^{-7}}{2 \cdot 10^{-6}} (70 \cdot 0,025 - 0,775)} = \frac{775}{0,2925} = 2,65 \cdot 10^3 \text{ m}^2$$

e) The various flow configurations **4p**

Typical concentration profiles in membrane modules with different flow arrangements

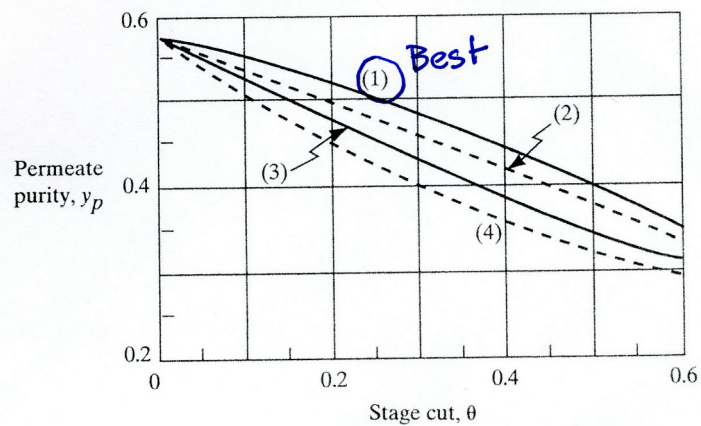
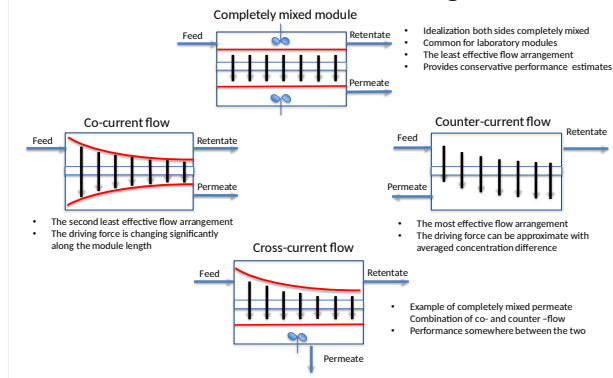


FIGURE 13.8-2. Effect of stage cut and flow pattern on permeate purity. Operating conditions for air are as follows: $x_f = 0.209$, $\alpha^* = 10$, $p_h/p_l = 380 \text{ cm Hg}/76 \text{ cm Hg} = 5$, $P'_A = 500 \times 10^{-10} \text{ cm}^3(\text{STP}) \cdot \text{cm/s} \cdot \text{cm}^2 \cdot \text{cm Hg}$. (1) countercurrent flow, (2) cross-flow, (3) cocurrent flow, (4) complete mixing (W5). [Reprinted from W. P. Walawender and S. A. Stern, *Sep. Sci.*, 7, 553 (1972). By courtesy of Marcel Dekker, Inc.]