Øving 8

1

Separate a feed gas that consists of CO_2 and methane with a cellulose acetate membrane that has effective membrane layer thickness of 1 μm. The feed gas flow rate is 1 mol/s and has 50 mol % of CO_2 and 50 mol % of methane at 35 °C. The membrane permeabilities were measured at 35°C:

P_{CO2} = 15.0 x 10^{-10} [cc(STP)cm]/[cm^2 s cm Hg] and P_{CH4} = 0.48 x 10^{-10} [cc(STP)cm]/[cm^2 s cm Hg]. A completely mixed membrane module is used with retentate side pressure p_r = 20 atm and permeate side pressure p_p =1.1 atm.

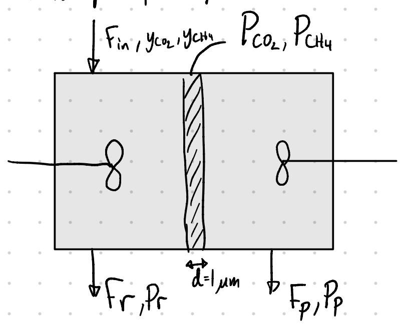
a) Determine the permeate and retentate purity for stage cuts: 0.25, 0.5, 0.75.

b) Evaluate the fluxes and the membrane area

c) Find the limit for maximum permeate purity

Note: You may use a numerical (Excel, Matlab), graphical or analytical solution.

Formler er hentet fra powerpoints.



La $A = CO_2$

a) Operating line: $$y_{Ar} = \frac{y_A - \theta\, y_{Ap}}{1-\theta}$$

Rate transfer: $$y_{Ar} = \frac{y_{Ap}\left[R(1-y_{Ap})(\alpha-1)+1\right]}{\alpha-(\alpha-1)\,y_{Ap}}$$

$$R = \frac{P_p}{P_r} = \frac{1{,}1}{20} = 0{,}055$$

$$\alpha = \frac{P_A}{P_{CH_4}} = \frac{15}{0{,}48} = 31{,}25$$

```python
import scipy.optimize as sco
import numpy as np
# Konstanter
R = 1.1/20
yA = 0.5
thetas = np.array([0, 0.25, 0.5, 0.75, 1])
P_A = 15*10**(-10)
P_B = 0.48*10**(-10)
alpha = P_A/P_B
L = 1
p_r = 20
p_p = 1.1
nF = 1
rho_A = 1/22400

def operating_line(y_ap):
    y_ar = (yA - stage_cut*y_ap) / (1- stage_cut)
    return y_ar

def rate_transfer(y_ap):
    y_ar = (y_ap*((R*(1-y_ap)*(alpha-1))+1)) / (alpha - (alpha-1)*y_ap)
    return y_ar

def solver(y_ap):
    return rate_transfer(y_ap) - operating_line(y_ap)
print("##################################")
for stage_cut in thetas:
    if stage_cut == 1:
        y_ap = yA
    else:
        ans = sco.root(solver, 0.9)
        y_ap = ans['x'][0]
    y_ar = rate_transfer(y_ap)
    print(f"For theta = {stage_cut}:")
    print(f"y_ar = {y_ar:.4f} og y_ap = {y_ap:.4f}")

    J_AV = 76/(10**(-4)) * P_A/L * (p_r*y_ar - p_p*y_ap)
    J_BV = 76/(10**(-4)) * P_B/L * (p_r*(1 - y_ar) - p_p*(1 - y_ap))
    print(f"J_AV = {J_AV:.5f} og y_BV = {J_BV:.6f}")

    A = (nF * stage_cut * y_ap)/(J_AV * rho_A) * 10**(-6)
    print(f"A = {A:.2f}")
    print("##################################")
```

Setter inn i python, løser med scipy.optimize.root

=> $\theta = 0{,}25$, $y_{Ar} = 0{,}3545$, $y_{Ap} = 0{,}9365$

$\theta = 0{,}50$, $y_{Ar} = 0{,}1718$, $y_{Ap} = 0{,}8282$

$\theta = 0{,}75$, $y_{Ar} = 0{,}0856$, $y_{Ap} = 0{,}6381$

b) $$J_{AV} = \frac{P_A}{L}\left(P_r y_{Ar} - P_p y_{Ap}\right)$$

$$J_{BV} = \frac{P_B}{L}\left(P_r(1-y_{Ar}) - P_p(1-y_{Ap})\right)$$

enheter: $$\frac{(cc\ (STP)\ cm)/(cm^2\ s\ cmHg)}{\mu m} \cdot atm$$

$$\frac{(cc\ (STP)\ cm)/(cm^2\ s\ cmHg)}{\mu m \cdot 10^{-4}\frac{cm}{\mu m}} \cdot atm \cdot \frac{76\ cmHg}{atm}$$

$$\boxed{\frac{76}{10^{-4}} \cdot \frac{cc\ (STP)}{cm^2 \cdot s}}$$

korrigerer med

Løser direkte i python

$\Rightarrow$ $\theta = 0{,}25$, $J_{AV} = 0{,}00691$, $J_{BV} = 0{,}000468$

$\theta = 0{,}50$, $J_{AV} = 0{,}00288$, $J_{BV} = 0{,}000597$

$\theta = 0{,}75$, $J_{AV} = 0{,}00115$, $J_{BV} = 0{,}000653$

Finner arealet ved:

$$J_{An} = \rho_{AV} \cdot J_{AV}, \quad J_{An} = \frac{\dot{n}_{Ap}}{A} = \frac{\dot{n}_p y_{Ap}}{A} = \frac{\dot{n}_F \cdot \theta \cdot y_{Ap}}{A}$$

$$\Rightarrow A = \frac{\dot{n}_F \cdot \theta \cdot y_{Ap}}{J_{AV} \cdot \rho_{AV}} \Rightarrow \left[\frac{mol/s}{\frac{cc\ (STP)}{cm^2 \cdot s}\ [?]}\right] = [cm^2]$$

ρ_{AV} er gitt i: $\frac{mol}{cc\ STP}$

Må finne ρ_{AV} med enheter $\frac{mol}{cc\ STP}$
se siste side

$$\Rightarrow \rho_{AV} = \frac{1\ mol}{22400\ cc\ STP}$$

Løser i python:

$\theta = 0{,}25$, $A = 75{,}9\ m^2$

$\theta = 0{,}50$, $A = 322\ m^2$

$\theta = 0{,}75$, $A = 931\ m^2$

c) Dette er når $\theta = 0$, da er operating line vertikal,
Fra python, er da:

$\theta = 0$, $y_{Ap,max} = 0{,}9656$

2

A cellulose-acetate membrane with an area of 4.0 x 10^{-3} m^2 is used at 25 °C to determine the permeability constants for reverse osmosis of a feed salt solution containing 12.0 kg NaCl/m^3 (ρ = 1005.5 kg/m^3). The product solution has a concentration of 0.468 kg NaCl/m^3 (ρ = 997.3 kg/m^3). The measured product flow rate is 3.84 x 10^{-8} m^3/s and the pressure difference used is 56.0 atm. Calculate the permeance constant and the solute rejection R. (Geankoplis, 4th ed.)

Osmotic Pressure of Various Aqueous Solutions at 25 °C.

Sodium Chloride Solutions			Sea Salt Solutions		Sucrose Solutions	
$\frac{mol\ NaCl}{kg\ H_2O}$	Density (kg/m^3)	Osmotic Pressure (atm)	Wt.% Salts	Osmotic Pressure (atm)	Solute Mol.Frac. $x10^{-3}$	Osmotic Pressure (atm)
0	997.0	0	0	0	0	0
0.01	997.4	0.47	1.00	7.10	1.798	2.48
0.10	1001.1	4.56	3.45	25.02	5.375	7.48
0.50	1017.2	22.55	7.50	58.43	10.69	15.31
1.00	1036.2	45.80	10.00	82.12	17.70	26.33
2.00	1072	96.2				

$\Delta p = 56$ atm , $\varphi_p = 3{,}84 \cdot 10^{-8}$ m^3/s , $A = 4 \cdot 10^{-3}$ m^2

$C_F = 12{,}0$ kg/m^3 , $C_p = 0{,}468$ kg/m^3

$\rho_F = 1005{,}5$ kg/m^3 , $\rho_p = 997{,}3$ kg/m^3

Produktstrømmen er fortynnet, antar $\rho_w = \rho_p = 997{,}3$ kg/m^3

Finner fluksen av vann og salt fra produktstrømmen

$$N_w = \frac{\varphi_p \cdot \rho_w}{A} = \frac{3{,}84 \cdot 10^{-8}\ m^3/s \cdot 997{,}3\ kg/m^3}{4 \cdot 10^{-3}\ m^2} = 9{,}57 \cdot 10^{-3}\ \frac{kg}{m^2 \cdot s}$$

S = Salt

$$N_S = \frac{\varphi_p \cdot C_p}{A} = \frac{3{,}84 \cdot 10^{-8}\ m^3/s \cdot 0{,}468\ kg/m^3}{4 \cdot 10^{-3}\ m^2} = 4{,}49 \cdot 10^{-6}\ \frac{kg}{m^2 \cdot s}$$

Feed: $m_{NaCl} = 12$ kg/m^3

$m_w = (1005{,}5 - 12)$ kg/m^3 $= 993{,}5$ kg/m^3

$$\frac{n_{NaCl}}{m_w} = \frac{\left(\frac{12\ kg/m^3}{58{,}44\ g/mol}\right)}{993{,}5\ kg/m^3} = \underline{0{,}2067\ mol\ NaCl/kg\ H_2O}$$

Ønsker Π_F, Interpolerer tabellen

$$a = \frac{22,55 - 4,56}{0,5 - 0,1} = 44,975$$

$$y - 4,56 = 44,975(x - 0,1)$$

La $x = 0,2067$

$$\Rightarrow \Pi_F = y = 4,56 + 44,975(0,2067 - 0,1) = \underline{9,36 \text{ atm}}$$

For produktstrømmen

$$m_{NaCl} = 0,468 \text{ kg/m}^3$$

$$m_w = (997,3 - 0,468) \text{ kg/m}^3 = 996,832 \text{ kg/m}^3$$

$$\frac{n_{NaCl}}{m_w} = \frac{\left(\frac{0,468 \text{ kg/m}^3}{58,44 \text{ g/mol}}\right)}{996,832 \text{ kg/m}^3} = \underline{8,034 \cdot 10^{-3} \text{ mol NaCl / kg } H_2O}$$

Interpoleres igjen: $\Rightarrow \underline{\Pi_P = 0,38 \text{ atm}}$

$$\Delta\Pi = 9,36 - 0,38 = 8,98 \text{ atm}$$

$$a_w = \frac{N_w}{\Delta P - \Delta\Pi} = \frac{9,57 \cdot 10^{-3} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}}{(56 - 8,98) \text{ atm}} = \underline{\underline{2,04 \cdot 10^{-4} \frac{\text{kg}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}}}}$$

$$a_s = \frac{N_s}{\Delta C} = \frac{4,49 \cdot 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}}{12 \frac{\text{kg}}{\text{m}^3} - 0,468 \frac{\text{kg}}{\text{m}^3}} = \underline{\underline{3,89 \cdot 10^{-7} \text{ m/s}}}$$

$$\underline{\underline{R = 1 - \frac{C_P}{C_F} = 1 - \frac{0,468}{12} = 0,961}}$$

Appendix

ρ_{AV}. Er molar tetthet => $\frac{n}{V}$, hvor vi dette tilfellet trenger enheten cc STP for volumet

Dermed har vi: $\rho_{AV} = \frac{n}{V}$

Bruker vi ideell gasslov, så blir: $pV = nRT \Rightarrow \frac{n}{V} = \frac{P}{RT}$

Siden cc STP = cm^3 ved STP, så setter vi inn STP i uttrykket:

$$\rho_{AV} = \frac{n}{V} = \frac{1 \cancel{atm}}{0{,}0821 \frac{\cancel{atm}\cdot L}{mol\cdot \cancel{K}} \cdot 273 \cancel{K}} \approx \frac{1}{22{,}4\ L/mol}$$

For de som ikke kjenner igjen dette tallet, så er det molare volumet til gass ved STP, $V_m = \frac{V}{n} = 22{,}4\ L/mol$

Per definisjon, siden $\rho_{AV} = \frac{n}{V}$ og $V_m = \frac{V}{n}$ $\Rightarrow \rho_{AV} = \frac{1}{V_m}$

(Men det trengs ikke i oppgaven)

Men vi mangler fortsatt å få cc STP enheten, siden vi fant ρ_{AV} ved STP, så er:

$$\rho_{AV} = \frac{1}{22{,}4\ L/mol} \text{ (ved STP)} = \frac{1\ mol}{22{,}4\ L\ STP}$$

Litt enhetskonvertering:

1 cc STP = 1 cm^3 STP

1 cm^3 STP = 10^{-3} dm^3 STP

1 dm^3 STP = 1 L STP

$$\Rightarrow 1\ L\ STP = 1 \cancel{L\ STP} \cdot \frac{1 \cancel{dm^3\ STP}}{1 \cancel{L\ STP}} \cdot \frac{10^3 \cancel{cm^3\ STP}}{1 \cancel{dm^3\ STP}} \cdot \frac{1\ cc\ STP}{1 \cancel{cm^3\ STP}}$$

$$\Rightarrow 1\ L\ STP = 1000\ cc\ STP$$

Dermed:

$$\rho_{AV} = \frac{1\ mol}{22{,}4\ L\ STP} = \frac{1\ mol}{22400\ cc\ STP}$$