

Øving 4

1

In a heat exchanger with constant heat capacities and constant UA , we have the following simple formula for the total heat transfer from the hot to the cold stream [J/s]

$$Q = UA\Delta T_M$$

where $\Delta T_M = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$ is the log-mean temperature difference.

Countercurrent absorption is similar to a heat exchanger, but the driving force is the concentration difference between y in the gas phase and $y^*(x)$ in equilibrium with the liquid phase. With constant slopes for the equilibrium and operating lines (which is reasonable for dilute mixtures), the total mass transfer of A from the gas to liquid stream is [mol A/s] (as derived in the lecture)

$$\bar{N}_A = K_y(aSv)(y - y^*)M$$

where $(aSv) [\text{m}^2]$ is the total mass transfer area inside the column and $(y - y^*)M$ is the log mean vapor concentration difference between the phases, $(y - y^*)_M = \frac{(y-y)_1 - (y-y)_2}{\ln((y-y^*)_1 / (y-y^*)_2)}$ ($1=\text{top of column}$, $2=\text{btm of column}$).

*it should be easy to remember this formula at the exam and possibly for the rest of your life!

We want to apply this easy-to-remember formula to calculate the absorption of SO_2 using water. A tray tower is to be designed to absorb SO_2 (A) from an air stream by using pure water at 293 K (680F). The entering gas contains 20 mol % SO_2 , and that leaving 2 mol % at a total pressure of 101.3 kPa. The inert air flow rate is 150 kg air/h \cdot m², and the entering water flow rate is 6000 kg water/h \cdot m². Assuming an overall tray efficiency of 25%, how many theoretical trays and actual trays are needed? Assume that the tower operates at 293 K (20°C). It is not really a dilute mixture, so we expect some error.

Given data: The mole fraction of SO_2 drops from 0.2 (btm) to 0.02 (top) in air. The airflow (inert) is $6.53 \cdot 10^{-4}$ kmol/s. The entering liquid is $4.20 \cdot 10^{-2}$ kmol/s of pure water. The cross-sectional area of the column is 0.0929 m².

$$L_0 = 4.20 \text{ kmol/s}$$

$$x_0 = 0$$

$$V_1 = ?$$

$$y_1 = 0.02$$

$$\text{top}$$

$$L_N = ?$$

$$x_N = ?$$

$$V_{NH} = 6.53 \cdot 10^{-4}$$

$$y_{NH} = 0.2$$

$$\text{bottom}$$

Find the total mass transfer of A from the vapor to liquid,

$$\bar{N}_A = y_{\text{btm}} V_{\text{btm}} - y_{\text{top}} V_{\text{top}} [\text{mol A/s}]$$

Oppgitt: Column efficiency $= \frac{N_t - 1}{N_a} = 0.25$

$$A = 0.0929 \text{ m}^2$$

$$V_{N+1} = \frac{V'_1}{1-0.2} = 8.1625 \cdot 10^{-4} \text{ kmol/s}$$

$$V_1 = \frac{V'_1}{1-0.02} = 6.6632 \cdot 10^{-4} \text{ kmol/s}$$

$$\Rightarrow N_A = 0,2 \cdot 6,1625 \cdot 10^{-4} - 0,02 \cdot 6,632 \cdot 10^{-4} \text{ kmol/s}$$

$$\underline{N_A = 1,499 \cdot 10^{-4} \text{ kmol/s}}$$

(ii)

The column operates at 1 atm and 20C. Use the equilibrium data to find the slope m of the equilibrium line for the concentration range of interest.

Linearregresjon av dataen gir:

$$y = 31,62 \cdot X - 0,01$$

$$\Rightarrow \underline{m = 31,62}$$

A-3-19 Equilibrium Data for SO₂-Water System

Mole Fraction SO ₂ in Liquid, x_A	Partial Pressure of SO ₂ in Vapor, y_A (mm Hg)		Mole Fraction SO ₂ in Vapor, y_A ; P = 1 Atm	
	20°C (293 K)	30°C (303 K)	20°C	30°C
0	0	0	0	0
0.000562	0.5	0.6	0.000658	0.000790
0.0001403	1.2	1.7	0.00158	0.00223
0.000203	3.2	4.7	0.00421	0.00619
0.000422	5.8	8.1	0.00763	0.01065
0.000564	8.5	11.8	0.01120	0.0155
0.000842	14.1	19.7	0.01855 $\sim 0,02$	0.0259
0.001403	26.0	36	0.0342	0.0473
0.001965	39.0	52	0.0513	0.0685
0.00279	59	79	0.0775	0.1040
0.00420	92	125	0.121	0.1615
0.00698	161	216	0.212 $\sim 0,2$	0.284
0.01385	336	452	0.443	0.594
0.0206	517	688	0.682	0.905
0.0273	698		0.917	

Source: T. K. Sherwood, *Ind. Eng. Chem.*, 17, 745 (1925).

(iii)

Compute K_{ya} using the data $k_{xa} = 0.85 \text{ kmol/s.m}^3$ and $k_{ya} = 0.037 \text{ kmol/s.m}^3$

$$\frac{1}{K_{ya}} = \frac{1}{k_{ya}} + \frac{m}{k_{xa}}$$

$$\Rightarrow K_{ya} = \frac{1}{\frac{1}{k_{ya}} + \frac{m}{k_{xa}}} = \frac{1}{\frac{1}{0.037} + \frac{31,62}{0.85}} = \underline{0,016 \text{ kmol/s.m}^3}$$

(iv)

Find the logarithmic mean driving force $(y - y^*)_M$

$$(y - y^*)_M = \frac{(y_1 - y^*)_1 - (y_2 - y^*)_2}{\ln \frac{(y_1 - y^*)_1}{(y_2 - y^*)_2}} = \frac{(0,02 - 0) - (0,2 - 0,1)}{\ln \frac{0,02}{0,1}} = \underline{0,05}$$

$$y_1 = y_{top} = 0,02$$

$$y_2 = y_{bottom} = 0,2$$

$$X_1 = X_{top} = 0 \Rightarrow y_1^* = 0$$

y_2^* finnes ved interpolasjon fra tabellen, trange X_N , mdbalanse SO₂:

$$X_N = \frac{N_A}{L_0 + N_A} = \underline{0,00356}$$

$$y_2^* = 0,0775 + (0,00356 - 0,00275) \frac{0,121 - 0,0775}{0,0642 - 0,00275}$$

$$y_2^* = 0,10$$

(v)

Use the easy-to-remember formula to find the packing height z .

$$N_A = K_{ya} S_z (y - y^*)_M \Rightarrow z = \frac{N_A}{K_{ya} S(y - y^*)_M} = \frac{1,499 \cdot 10^{-4} \text{ kmol/s}}{0,016 \text{ kmol/s.m}^3 \cdot 0,05 \cdot 0,05}$$

$$\underline{z = 2,02 \text{ m}}$$

Absorption of CO₂ in water-amine solution (part 2)

This continues the previous exercise, but now you should use the rate-based method (mass transfer) rather than equilibrium stages. Note that since we do not really have stages anymore, we choose to use a different notation with "top" and "btm", so x_0 is now x_{top} , etc. (see Figure).

Let us first repeat the basic data for the column:

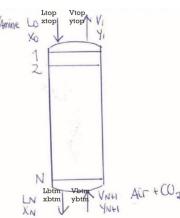
$$V_{btm} = 22 \text{ kmol/s}$$

$$y_{btm} = 0.1176 \text{ (CO}_2 \text{ mol fraction) (feed)}$$

$$y_{top} = 0.0116$$

$$x_{top} = 0.0219 \text{ (feed)}$$

$$x_{btm} = 0.0532$$



Assuming constant inert flows for air (V') and water-amine (L') through the column, we have

$$V' = V_{btm} (1 - y_{btm}) = 19.41 \text{ kmol/s}$$

$$L' = 69.80 \text{ kmol/s}$$

The operating line is not quite linear, but it goes through the points (x_{top}, y_{top}) and (x_{btm}, y_{btm}) . To simplify, you may assume that the operating line is a straight line between these two points

$$\frac{L'}{V'} = \frac{y_{top} - y_{btm}}{x_{top} - x_{btm}}$$

(which corresponds to using a constant

$$\Rightarrow p = 1.2 \text{ bar}, T = 60^\circ\text{C}$$

Table: Equilibrium data for CO₂ and water-MEA at 1.2 bar and about 60°C (inert air present).

$x_{CO_2}(\%)$	$y_{CO_2}(\%)$
2.227	1.162
3.001	2.917
3.339	3.783
4.101	6.057
4.700	8.448
5.324	11.60

The new part of the problem:

The overall mass transfer rate referred to the gas side is with pressure as the driving force (A=CO₂):

$$N_A = K_G \cdot a (p_A - p^* \cdot a)$$

where $p^* \cdot a$ is the partial pressure of A that would result if we had gas in equilibrium with the liquid. The following data is given:

$$K_G \cdot a = 12.5 \text{ lbmol/(h * ft}^3 * \text{atm)}$$

- a) Use the data to find $K_g \alpha$ [kmol/(s*m³ packing)] as used in the book,
 $N_A = K_g \alpha (y_A - y_A^*)$

where y_A^* is the mole vapor mole fraction of a gas (imagined) in equilibrium with the liquid (which has mole fraction x_A).

a) Setze $N_A = N_A \Rightarrow K_g \alpha (y_A - y_A^*) = K_g \alpha (p_A - p_A^*)$

$$K_g \alpha = \frac{K_g \alpha (p_A - p_A^*)}{(y_A - y_A^*)}, \quad P_A = P \cdot y_A$$

$$\Rightarrow K_g \alpha = K_g \alpha \frac{P(y_A - y_A^*)}{(y_A - y_A^*)}$$

$$K_g \alpha = 12.5 \frac{1 \text{ b.mol}}{\text{h} \cdot \text{ft}^3 \cdot \text{atm}} \cdot 1.2 \text{ bar}$$

$$= 15 \cdot \frac{1 \text{ b.mol} \cdot \text{bar}}{\text{h} \cdot \text{ft}^3 \cdot \text{atm}} \cdot 1.01325 \frac{\text{atm}}{\text{bar}} \cdot 0.45 \frac{\text{kmol}}{1 \text{ b.mol}} \cdot \frac{1}{3600 \text{ s}} \cdot \frac{1}{(0.3048 \text{ m})^3}$$

$$\underline{K_g \alpha = 0.067 \frac{\text{kmol}}{\text{s} \cdot \text{m}^3}}$$

- b) Find the required packing height when $S = 253.5 \text{ m}^2$ (cross-sectional area of column) and you can assume constant molar flows in the column.

Note from the McCabe-Thiele diagram from the previous exercise that the equilibrium line is far from linear, so we cannot use the simple method from Problem 1.

(Hint: Make a table of $1/(y_A - y_A^*)$ as a function of y_A ; you can then integrate graphically, for example, by counting the number of squares on an mm-paper).

$$\text{b) Massebalanse gtr: } N_A dA = d(V y_A) \quad V = \text{const}$$

$$K_A (y_A - y_A^*) S dA = V dy_A$$

$$\Rightarrow Z = \frac{V}{K_A S} \int_{y_{\text{top}}}^{y_{\text{btm}}} \frac{dy}{y_A - y_A^*} = \frac{20,82 \text{ kmol/s}}{0,067 \text{ kmol/s.m}^2 \cdot 253,5 \text{ m}^2} \int_{y_{\text{top}}}^{y_{\text{btm}}} \frac{dy}{y_A - y_A^*}$$

Operating line: (fra fornise øving)

$$y_{N+1} = \frac{L_N}{V_{N+1}} x_N + \frac{y_1 V_1 - x_0 L_0}{V_{N+1}}$$

$$y = \frac{L}{V} x + y_{\text{top}} - \frac{L}{V} x_{\text{top}}$$

$$y = \frac{L}{V} (x - x_{\text{top}}) + y_{\text{top}}, \text{ gitt: } \frac{L}{V} = \frac{y_{\text{top}} - y_{\text{btm}}}{x_{\text{top}} - x_{\text{btm}}} = \frac{0,0116 - 0,01176}{0,0219 - 0,0532}$$

$$y = 3,387 x - 0,0626$$

$$\frac{L}{V} = 3,387$$

Før denne tabellen:

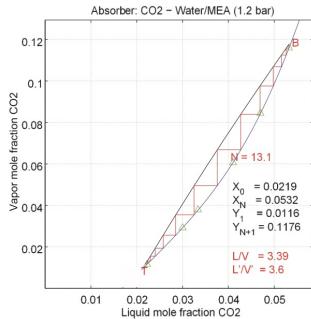
Gitt i oppgaven
↓
↓
Operating line

x_A	y_A^*	y_A	$1/(y_A - y_A^*)$
0,02227	0,01162	0,01282849	827,47892
0,03001	0,02917	0,03904387	101,277412
0,0339	0,03783	0,05049193	78,97690163
0,04101	0,06057	0,07630087	63,56927494
0,047	0,08448	0,096589	82,58320258
0,05324	0,116	0,11772388	580,086781

$$Z = 1,226 \int_{y_{\text{top}}=0,0116}^{y_{\text{btm}}=0,01176} \frac{dx}{y_A - y_A^*} = 1,226 \cdot 13,5$$

diagram nede side

$$\underline{\underline{Z = 16,6 \text{ m}}}$$



Lar $V = \frac{V_{\text{top}} + V_{\text{btm}}}{2}$

$$V = \frac{19,64 + 22}{2} \text{ kmol/s}$$

fornise øving

$$V = 20,82 \text{ kmol/s}$$

