Øving 3 - Matematikk 4N

Oppgave 1 a) Lager "Composite rule": $(a = x_0,\ b = x_m)$

$$\Rightarrow \int_a^b f(x)dx = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \dots + \int_{x_{m-1}}^{x_m} F(x)dx$$

Ved simpsons regel:

$$\int_a^b f(x)dx \approx \frac{h}{6}\left(f(x_0) + 4f(x_{1/2}) + f(x_1)\right) + \frac{h}{6}\left(f(x_1) + 4f(x_{3/2}) + f(x_2)\right)$$

$$+ \dots + \frac{h}{6}\left(f(x_{m-1}) + 4f(x_{m-1/2}) + f(x_m)\right)$$

Ved å trekke sammen, og å inkludere første ledd i $S[f](x_2, x_3)$ og siste ledd i $S[f](x_{m-2}, x_{m-1})$:

$$\int_a^b f(x) \approx \frac{h}{6}\left(f(x_0) + 4f(x_{1/2}) + 2f(x_1) + 4f(x_{3/2}) + 2f(x_2) + \dots\right.$$
$$\left. + 2f(x_{m-1}) + 4f(x_{m-1/2}) + f(x_m)\right) \quad \underline{\underline{QED}}$$

c) Ettersom $CSR[f]$ er en sum av $S[f]$, med absoluttverdi av feil $\frac{(b-a)^5}{2880} f^{(4)}(\xi)$, vil vi få:

$$\boxed{x_i - x_{i-1} = \frac{b-a}{m}}$$

$$|I[f] - CSR[f]| \leq \left|\sum_{i=1}^{m} \frac{(x_i - x_{i-1})^5}{2880} f^{(4)}(\xi)\right| = \left|\sum_{i=1}^{m} \frac{(b-a)^5}{m^5} \cdot \frac{1}{2880} f^{(4)}(\xi)\right|$$

$f^{(4)}(\xi)$ er ulik for alle delintervallene, men $\leq M_4$ i alle delintervall:

$$\Rightarrow |I[f] - CSR[f]| \leq \frac{(b-a)^5}{m^5} M_4 \sum_{i=1}^{m} \frac{1}{2880} = \frac{(b-a)^5}{m^5} M_4 \cdot \frac{m}{2880}$$

$$= \frac{M_4}{2880} \cdot \frac{(b-a)^5}{m^4} = \frac{M_4}{2880} h^4 (b-a)$$

$$\Rightarrow \underline{\underline{|I[f] - CSR[f]| \leq \frac{M_4}{2880} \frac{(b-a)^5}{m^4} = \frac{M_4}{2880} h^4 (b-a)}} \quad QED$$

Oppgave 2

a) $P_0 = 1$, $P_1 = x - \frac{3}{2}$, $\varphi_2 = x^2$, $\varphi_3 = x^3$

$$P_2 = x^2 - \frac{\langle P_1, \varphi_2 \rangle}{\|p_1\|^2} P_1 - \frac{\langle P_0, \varphi_2 \rangle}{\|p_0\|^2} P_0$$

$$= x^2 - \frac{27/4}{9/4}\left(x - \frac{3}{2}\right) - \frac{9}{3} \cdot 1$$

$$= x^2 - 3\left(x - \frac{3}{2}\right) - \frac{9}{3} \cdot 1$$

$$\underline{P_2 = x^2 - 3x + \frac{3}{2}}$$

$$P_3 = x^3 - \frac{\langle P_2, \varphi_3 \rangle}{\|p_2\|^2} P_2 - \frac{\langle P_1, \varphi_3 \rangle}{\|p_1\|^2} P_1 - \frac{\langle P_0, \varphi_3 \rangle}{\|p_0\|^2} P_0$$

$$= x^3 - \frac{243/40}{27/20}\left(x^2 - 3x + \frac{3}{2}\right) - \frac{729/40}{9/4}\left(x - \frac{3}{2}\right) - \frac{81/4}{3} \cdot 1$$

$$= x^3 - \frac{9}{2}\left(x^2 - 3x + \frac{3}{2}\right) - \frac{81}{10}\left(x - \frac{3}{2}\right) - \frac{27}{4}$$

$$\underline{= x^3 - \frac{9}{2}x^2 + \frac{27}{5}x - \frac{27}{20}}$$

d) $x_1 = \frac{3(5 - \sqrt{15})}{10}$, $x_2 = \frac{3}{2}$, $x_3 = \frac{3(5 + \sqrt{15})}{10}$

$$L_1 = \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} = \underline{\underline{\frac{10}{27}\left(x^2 - \left(3 + \frac{3\sqrt{15}}{10}\right)x + \frac{9(5 + \sqrt{15})}{20}\right)}}$$

$$L_2 = \frac{x - x_1}{x_2 - x_1} \cdot \frac{x - x_3}{x_2 - x_3} = \underline{\underline{-\frac{20}{27}\left(x^2 - 3x + \frac{9}{10}\right)}}$$

$$L_3 = \frac{x - x_1}{x_3 - x_1} \cdot \frac{x - x_2}{x_3 - x_2} = \underline{\underline{\frac{10}{27}\left(x^2 - \left(3 - \frac{3\sqrt{15}}{10}\right)x + \frac{9(5 - \sqrt{15})}{20}\right)}}$$

$$W_1 = \int_0^3 L_1(x)\,dx = \underline{\underline{\frac{5}{6}}}$$

$$W_2 = \int_0^3 L_2(x)\,dx = \underline{\underline{\frac{4}{3}}}$$

$$W_3 = \int_0^3 L_3(x)\,dx = \underline{\underline{\frac{5}{6}}}$$

e)

$\hat{x}_i$	$-\sqrt{\frac{3}{5}}$	0	$\sqrt{\frac{3}{5}}$
$\hat{w}_i$	$\frac{5}{9}$	$\frac{8}{9}$	$\frac{5}{9}$

Overfører ved $x_i = \frac{b-a}{2}\hat{x}_i + \frac{b+a}{2}$, $w_i = \frac{b-a}{2}\hat{w}_i$

x_i	$\frac{3(5-\sqrt{15})}{10}$	$\frac{3}{2}$	$\frac{3(5-\sqrt{15})}{10}$
w_i	$\frac{5}{6}$	$\frac{4}{3}$	$\frac{5}{6}$

Dette stemmer med de tidligere beregningene