c) Max feil med ekvidistante noder:

$$f(x)-p_n(x)=\frac{f^{(n+1)}(\xi(x))}{(n+1)!}\prod_{i=0}^{n}(x-x_i) \quad \xi(x)\in(a,b)$$

$$W(x)=\prod_{i=0}^{n}(x-x_i)$$

$$\Rightarrow |W(x)|\leq \frac{h^{n+1}}{4}\cdot n!$$

$$e_3(x)=f(x)-p_3(x)=\frac{f^{(4)}(\xi(x))}{(n+1)!}W(x)$$

$$|e_3(x)|\leq \frac{h^4}{4\cdot 4}\cdot \max_{x\in[a,b]}|f^{(4)}(x)|$$

$$h=\frac{3-0}{3}=\frac{3}{3}=1$$

$$f^{(4)}=\frac{d^4}{dx^4}f(x)=(-1)^{4/2}(x^2\sin(x)-2\cdot 4x\cos(x)-4(4-1)\sin(x))$$

partall Hint

$$\underline{f^{(4)}(x)=x^2\sin(x)-8x\cos(x)-12\sin(x)}$$

$$=(x^2-12)\sin(x)-8x\cos(x)$$

x^2 har høyere grad enn $4x$

$$\max_{x\in[0,3]}|f^{(4)}(x)|=\max_{x\in[0,3]}|(x^2-12)|\cdot\max_{x\in[0,3]}|\sin(x)|+\max_{x\in[0,3]}|8x\cdot\cos(x)|$$

Her bør man dele i intervaller, men valgte å spare plass

$$=12+8\cdot 3=36$$

$$\max_{x\in[a,b]}|f^{(4)}(x)|=36$$

$$\Rightarrow \underline{\underline{|e_3(x)|\leq\frac{1}{16}\cdot 36=\frac{9}{4}\approx 2{,}25}}$$

Max feil Chebyshev noder

$$|e_3(x)| \leq \frac{(3-0)^4}{2^7 \cdot 4!} \max_{x \in [a,b]} |f^{(4)}(x)|$$

$$|e_3(x)| \leq \frac{3^4 \cdot 36}{2^7 \cdot 4!}$$

$$\underline{\underline{|e_3(x)| \leq \frac{243}{256} \approx 0{,}95}}$$

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