Oppgave 3 $f(x) = x^2 \sin(x)$

a)

x_i	0	1	2	3
$f(x_i)$	0	$\sin(1)$	$4\sin(2)$	$9\sin(3)$

$$l_0(x) = \frac{(x-1)(x-2)(x-3)}{(-1)\cdot(-2)\cdot(-3)} = -\frac{1}{6}(x^3 - 6x^2 + 11x - 6)$$

$$l_1(x) = \frac{(x-0)(x-2)(x-3)}{1\cdot(-1)\cdot(-2)} = \frac{1}{2}(x^3 - 5x^2 + 6x)$$

$$l_2(x) = \frac{(x-0)(x-1)(x-3)}{2\cdot 1\cdot(-1)} = -\frac{1}{2}(x^3 - 4x^2 + 3x)$$

$$l_3(x) = \frac{(x-0)(x-1)(x-2)}{3\cdot 2\cdot 1} = \frac{1}{6}(x^3 - 3x^2 + 2x)$$

$$P_3(x) = 0\cdot l_0(x) + \sin(1)\cdot l_1(x) + 4\sin(2)\cdot l_2(x) + 9\sin(3)\cdot l_3(x)$$

$$P_3(x) = \left(\frac{9}{6}\sin(3) - 2\sin(2) + \frac{1}{2}\sin(1)\right)x^3 + \left(-\frac{9}{2}\sin(3) + 8\cdot\sin(2) - \frac{5}{2}\sin(1)\right)x^2 + \left(3\sin(3) - 6\sin(2) + 3\sin(1)\right)x$$

$$P_3(x) \approx -1{,}186x^3 + 4{,}536x^2 - 2{,}508x$$

b) 4 noder $\Rightarrow n+1 = 4$, $n = 3$

$$\tilde{x}_i = \cos\left(\frac{(2i+1)\pi}{2(4)}\right), \quad i = 0, \ldots, n \qquad [-1, 1]$$

$$\Rightarrow \tilde{x}_i \;\left|\; \cos\left(\frac{\pi}{8}\right) \;\right|\; \cos\left(\frac{3\pi}{8}\right) \;\left|\; \cos\left(\frac{5\pi}{8}\right) \;\right|\; \cos\left(\frac{7\pi}{8}\right)$$

For $[a, b]$:

$$x_i = \frac{b-a}{2}\tilde{x}_i + \frac{b+a}{2}, \quad i = 0, \ldots, n$$

$$\Rightarrow x_i = \frac{3}{2}\tilde{x}_i + \frac{3}{2}, \quad i = 0, \ldots, n$$

$$x_i \;\left|\; \frac{3}{2}\cos\left(\frac{\pi}{8}\right) + \frac{3}{2} \;\right|\; \frac{3}{2}\cos\left(\frac{3\pi}{8}\right) + \frac{3}{2} \;\left|\; \frac{3}{2}\cos\left(\frac{5\pi}{8}\right) + \frac{3}{2} \;\right|\; \frac{3}{2}\cos\left(\frac{7\pi}{8}\right)$$