4.9C Heat-Exchanger Effectiveness

1. Introduction. In the preceding section the log mean temperature difference was used in the equation $q = UA\,\Delta T_{\text{lm}}$ in the design of heat exchangers. This form is convenient when the inlet and outlet temperatures of the two fluids are known or can be determined by a heat balance. Then the surface area can be determined if U is known. However, when the temperatures of the fluids leaving the exchanger are not known and a given exchanger is to be used, a tedious trial-and-error procedure is necessary. To solve these cases, a method called the heat-exchanger effectiveness ε is used which does not involve any of the outlet temperatures.

The heat-exchanger effectiveness is defined as the ratio of the actual rate of heat transfer in a given exchanger to the maximum possible amount of heat transfer if an infinite heat-transfer area were available. The temperature profile for a counterflow heat exchanger is shown in Fig. 4.9-6.

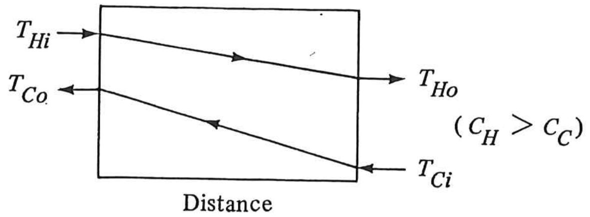

FIGURE 4.9-6. *Temperature profile for countercurrent heat exchanger.*

2. *Derivation of effectiveness equation.* The heat balance for the cold (C) and hot (H) fluids is

$$q = (mc_p)_H(T_{Hi} - T_{Ho}) = (mc_p)_C(T_{Co} - T_{Ci}) \tag{4.9-7}$$

Calling $(mc_p)_H = C_H$ and $(mc_p)_C = C_C$, then in Fig. 4.9-6, $C_H > C_C$, and the cold fluid undergoes a greater temperature change than the hot fluid. Hence, we designate C_C as $C_{\min}$ or minimum heat capacity. Then, if there is an infinite area available for heat transfer, $T_{Co} = T_{Hi}$. Then the effectiveness ε is

$$\varepsilon = \frac{C_H(T_{Hi} - T_{Ho})}{C_C(T_{Hi} - T_{Ci})} = \frac{C_{\max}(T_{Hi} - T_{Ho})}{C_{\min}(T_{Hi} - T_{Ci})} \tag{4.9-8}$$

If the hot fluid is the minimum fluid, $T_{Ho} = T_{Ci}$, and

$$\varepsilon = \frac{C_C(T_{Co} - T_{Ci})}{C_H(T_{Hi} - T_{Ci})} = \frac{C_{\max}(T_{Co} - T_{Ci})}{C_{\min}(T_{Hi} - T_{Ci})} \tag{4.9-9}$$

In both equations the denominators are the same and the numerator gives the actual heat transfer:

$$q = \varepsilon C_{\min}(T_{Hi} - T_{Ci}) \tag{4.9-10}$$

Note that Eq. (4.9-10) uses only inlet temperatures, which is an advantage when inlet temperatures are known and it is desired to predict the outlet temperatures for a given existing exchanger.

For the case of a single-pass, counterflow exchanger, combining Eqs. (4.9-8) and (4.9-9),

$$\varepsilon = \frac{C_H(T_{Hi} - T_{Ho})}{C_{\min}(T_{Hi} - T_{Ci})} = \frac{C_C(T_{Co} - T_{Ci})}{C_{\min}(T_{Hi} - T_{Ci})} \tag{4.9-11}$$

We consider first the case when the cold fluid is the minimum fluid. Rewriting Eq. (4.5-25) using the present nomenclature,

$$q = C_C(T_{Co} - T_{Ci}) = UA\frac{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})}{\ln[(T_{Ho} - T_{Ci})/(T_{Hi} - T_{Co})]} \tag{4.9-12}$$

Combining Eq. (4.9-7) with the left side of Eq. (4.9-11) and solving for T_{Hi},

$$T_{Hi} = T_{Ci} + \frac{1}{\varepsilon}(T_{Co} - T_{Ci}) \tag{4.9-13}$$

Subtracting T_{Co} from both sides,

$$T_{Hi} - T_{Co} = T_{Ci} - T_{Co} + \frac{1}{\varepsilon}(T_{Co} - T_{Ci}) = \left(\frac{1}{\varepsilon} - 1\right)(T_{Co} - T_{Ci}) \tag{4.9-14}$$

From Eq. (4.9-7) for $C_{\min} = C_C$ and $C_{\max} = C_H$,

$$T_{Ho} = T_{Hi} - \frac{C_{\min}}{C_{\max}}(T_{Co} - T_{Ci}) \tag{4.9-15}$$

This can be rearranged to give the following:

$$T_{Ho} - T_{Ci} = T_{Hi} - T_{Ci} - \frac{C_{\min}}{C_{\max}}(T_{Co} - T_{Ci}) \tag{4.9-16}$$

Substituting Eq. (4.9-13) into (4.9-16),

$$T_{Ho} - T_{Ci} = \frac{1}{\varepsilon}(T_{Co} - T_{Ci}) - \frac{C_{\min}}{C_{\max}}(T_{Co} - T_{Ci}) \tag{4.9-17}$$

Finally, substituting Eqs. (4.9-14) and (4.9-17) into (4.9-12), rearranging, taking the antilog of both sides, and solving for ε,

$$\varepsilon = \frac{1 - \exp\left[-\frac{UA}{C_{\min}}\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]}{1 - \frac{C_{\min}}{C_{\max}}\exp\left[-\frac{UA}{C_{\min}}\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]} \tag{4.9-18}$$

We define NTU as the number of transfer units as follows:

$$\text{NTU} = \frac{UA}{C_{\min}} \tag{4.9-19}$$

The same result would have been obtained if $C_H = C_{\min}$.

For parallel flow we obtain

$$\varepsilon = \frac{1 - \exp\left[\frac{-UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{1 + \frac{C_{\min}}{C_{\max}}} \tag{4.9-20}$$

In Fig. 4.9-7, Eqs. (4.9-18) and (4.9-20) have been plotted in convenient graphical form. Additional charts are available for different shell-and-tube and cross-flow arrangements (K1).

EXAMPLE 4.9-2. Effectiveness of Heat Exchanger

Water flowing at a rate of 0.667 kg/s enters a countercurrent heat exchanger at 308 K and is heated by an oil stream entering at 383 K at a rate of 2.85 kg/s ($c_p = 1.89$ kJ/kg · K). The overall $U = 300$ W/m^2 · K and the area $A = 15.0$ m^2. Calculate the heat-transfer rate and the exit water temperature.

Solution: Assuming that the exit water temperature is about 370 K, the c_p for water at an average temperature of (308 + 370)/2 = 339 K is 4.192 kJ/kg · K (Appendix A.2). Then, $(mc_p)_H = C_H = 2.85(1.89 \times 10^3) = 5387$ W/K and $(mc_p)_C = C_C = 0.667(4.192 \times 10^3) = 2796$ W/K $= C_{\min}$. Since C_C is the minimum, $C_{\min}/C_{\max} = 2796/5387 = 0.519$.

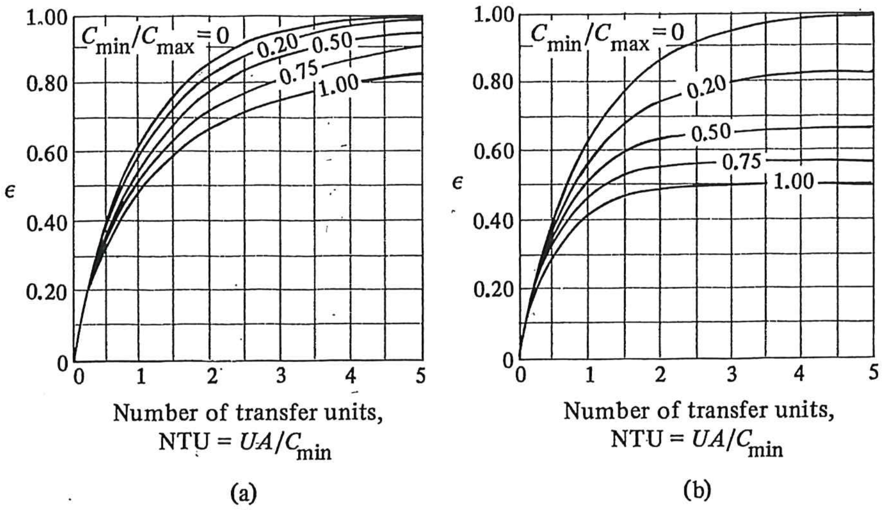

FIGURE 4.9-7. *Heat-exchanger effectiveness ε: (a) counterflow exchanger, (b) parallel flow exchanger.*

Using Eq. (4.9-19), NTU $= UA/C_{min} = 300(15.0)/2796 = 1.607$. Using Fig. (4.9-7a) for a counterflow exchanger, $\varepsilon = 0.71$. Substituting into Eq. (4.9-10),

$$q = \varepsilon C_{min}(T_{Hi} - T_{Ci}) = 0.71(2796)(383 - 308) = 148\,900 \text{ W}$$

Using Eq. (4.9-7),

$$q = 148\,900 = 2796(T_{Co} - 308)$$

Solving, $T_{Co} = 361.3$ K.