Innlevering 7 - Matematikk 3

Kapittel 13

Oppgave 1: a) Finner egenverdiene til matrisen. $A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$$\det(A - \lambda I_2) = \det\left(\begin{bmatrix} 2-\lambda & 3 \\ -1 & -2-\lambda \end{bmatrix}\right) = -4 + 2\lambda + 2\lambda + \lambda^2 + 3$$

$$= \lambda^2 - 1 = 0$$

(Bruker teorem 13,8)

$$\underline{\lambda = \pm 1}$$

Finner egenvektorene:

$\underline{\lambda = -1}$

$$A - I_2 = \begin{bmatrix} 1 & 3 \\ -1 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \cdot t}$$

$\underline{\lambda = 1}$

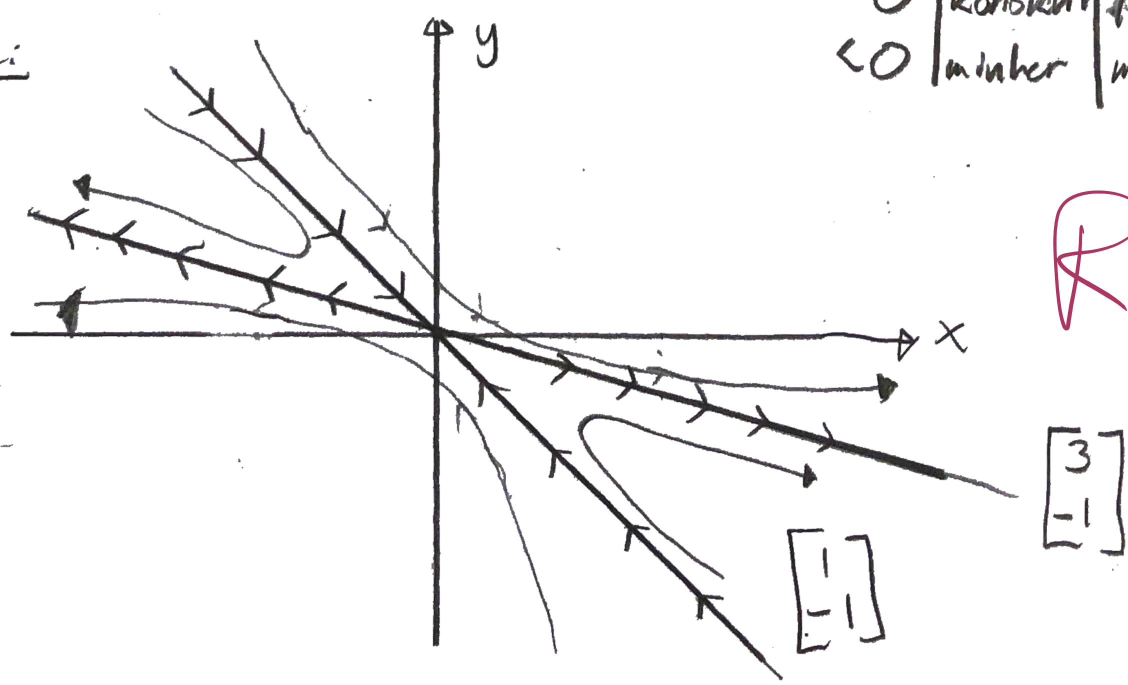
$$A + I_2 = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot t}$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}}}$$

Forklaring egenverdier:

λ	$e^{\lambda t}$	$V e^{\lambda t}$
>0	øker	fra origo
$=0$	konstant	i ro
<0	minker	mot origo

Skisse:



b) $A = \begin{bmatrix} 7 & -1 \\ 3 & 3 \end{bmatrix}$, egenverdier: $\det(A - \lambda I_2) = 0$

$$(7-\lambda)(3-\lambda) + 3 = 0$$
$$21 - 10\lambda + \lambda^2 + 3 = 0$$
$$\lambda^2 - 10\lambda + 24 = 0$$
$$(\lambda - 4)(\lambda - 6) = 0$$
$$\underline{\lambda = 4 \quad \vee \quad \lambda = 6}$$

Egenvektorer:
$\underline{\lambda = 4}$

$$A - 4\lambda = \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \cdot t}$$

$\underline{\lambda = 6}$

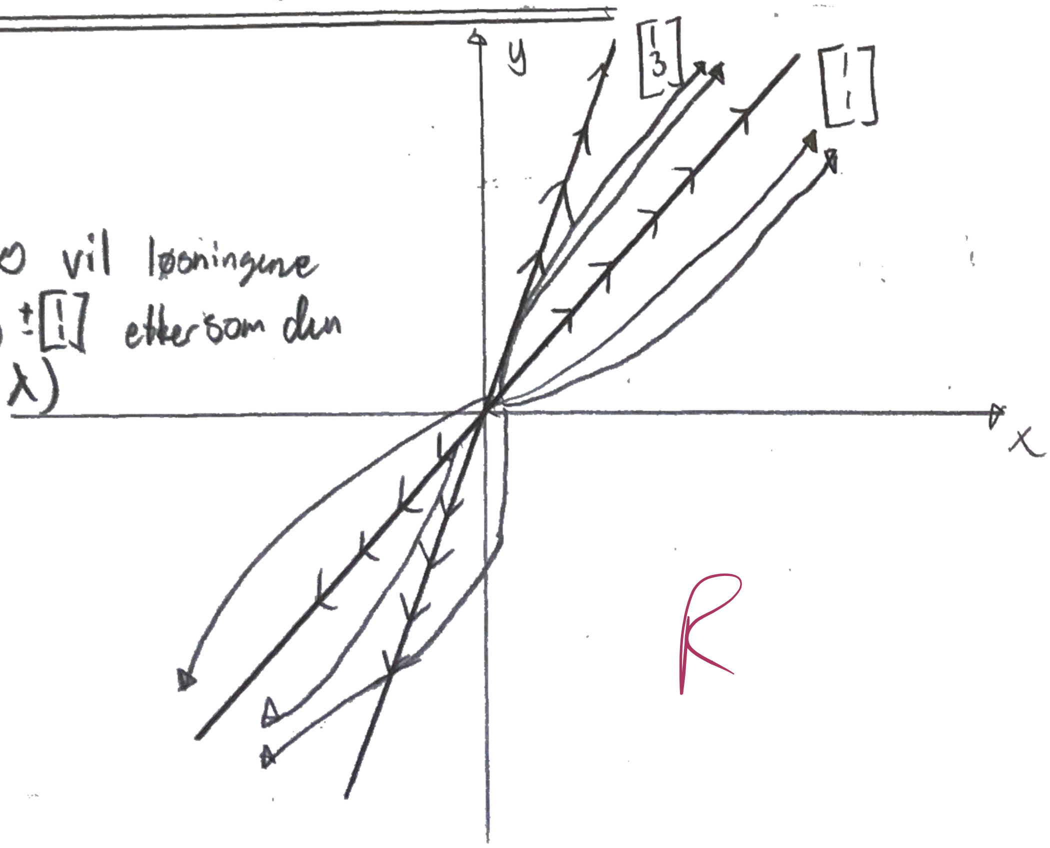
$$A - 6\lambda = \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot t}$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{6t}}}$$

R

Skisse:

(når $t \to \infty$ vil løsningene nærme seg $\pm\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ettersom den har størst λ)



R

c) $A=\begin{bmatrix}-3 & 2\\ -1 & -1\end{bmatrix}$ Egenverdier: $\det(A-\lambda I_2)=0$

$$(-3-\lambda)(-1-\lambda)+2=0$$
$$3+\lambda+3\lambda+\lambda^2+2=0$$
$$\lambda^2+4\lambda+5=0$$

$$\lambda=\frac{-4\pm\sqrt{4^2-4\cdot1\cdot5}}{2}=\underline{-2\pm i}$$

$\underline{\lambda=-2+i:}$

$$A-\lambda I_2=\begin{bmatrix}1-i & 2\\ -1 & 1-i\end{bmatrix}\sim\begin{bmatrix}1-i & 2\\ 0 & 0\end{bmatrix}\sim\begin{bmatrix}1 & -1+i\\ 0 & 0\end{bmatrix}\Rightarrow \underline{V_1=\begin{bmatrix}1-i\\ 1\end{bmatrix}}$$

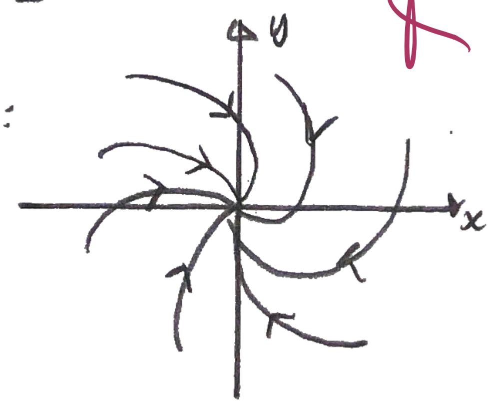
<u>teorem 13.16:</u>

eulers formel: $e^{\lambda t}=e^{(\alpha+\beta i)t}=e^{\alpha t}(\cos(\beta t)+i\cdot\sin(\beta t))$

$$\Rightarrow y(t)=c_1 e^{-2t}\left(\begin{bmatrix}1\\1\end{bmatrix}\cos(t)-\begin{bmatrix}-1\\0\end{bmatrix}\sin(t)\right)$$
$$+c_2 e^{-2t}\left(\begin{bmatrix}1\\1\end{bmatrix}\sin(t)+\begin{bmatrix}-1\\0\end{bmatrix}\cos(t)\right)$$
$$=e^{-2t}\left(c_1\begin{bmatrix}\cos t+\sin t\\ \cos t\end{bmatrix}+c_2\begin{bmatrix}\sin t-\cos t\\ \sin t\end{bmatrix}\right)$$
$$\Rightarrow y(t)=e^{-2t}\begin{bmatrix}\cos t+\sin t & -\cos t+\sin t\\ \cos t & \sin t\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}$$

R

$\alpha<0\Rightarrow$ inngående spiraler Skisse:



d) $A = \begin{bmatrix} -3 & -9 \\ 2 & 3 \end{bmatrix}$

$\underline{\lambda}$: $(-3-\lambda)(3-\lambda) + 18 = 0$

$-9 + \lambda^2 + 18 = 0$

$\lambda^2 = -3^2$

$\underline{\lambda = \pm 3i}$

Egenvektor: $\underline{\lambda = 3i}$

$$\begin{bmatrix} -3-3i & -9 \\ 2 & 3-3i \end{bmatrix} \sim \begin{bmatrix} -3-3i & -9 \\ 0 & 3-3i \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{3}{2} - i\frac{3}{2} \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{v} = \begin{bmatrix} -\frac{3}{2} + i\frac{3}{2} \\ 1 \end{bmatrix} t$$

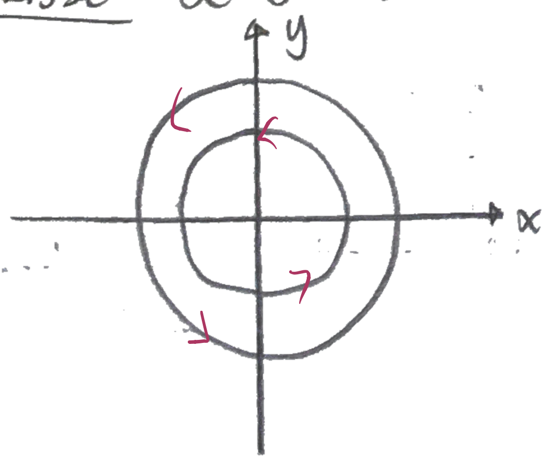
$\alpha = 0$

$$y(t) = \left(\cos(3t)\begin{bmatrix} 3/2 \\ 1 \end{bmatrix} - \sin(3t)\begin{bmatrix} 3/2 \\ 0 \end{bmatrix}\right)$$

$$+ \left(\sin(3t)\begin{bmatrix} 3/2 \\ 1 \end{bmatrix} + \cos(3t)\begin{bmatrix} 3/2 \\ 0 \end{bmatrix}\right)$$

$$\Rightarrow y(t) = \begin{bmatrix} \frac{3}{2}(\cos(3t) - \sin(3t)) & \frac{3}{2}(\cos(3t) + \sin(3t)) \\ \cos(3t) & \sin(3t) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

Skisse $\alpha = 0 \Rightarrow$ sirkulær



Aner ikke hvordan jeg skal bestemme retningen.

R

Oppgave 2:

a) $A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix}$

$$\det(A - \lambda I_3) = (2-\lambda)\begin{vmatrix} 3-\lambda & 5 \\ 0 & 5-\lambda \end{vmatrix}$$

$$= (2-\lambda)(3-\lambda)(5-\lambda)$$

$$\Rightarrow \underline{\lambda_1 = 2,\ \lambda_2 = 3,\ \lambda_3 = 5}$$

$\underline{\lambda_1 = 2:}$

$$A - 2I_3 = \begin{bmatrix} 0 & 3 & 5 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot t$$

$\underline{\lambda_2 = 3:}$

$$A - 3I_3 = \begin{bmatrix} -1 & 3 & 5 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{v_2} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \cdot t$$

$\lambda_3 = 5:$

$$A - 5I_3 = \begin{bmatrix} -3 & 3 & 5 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -3 & 0 & 25/2 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -25/6 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \underline{v_3} = \begin{bmatrix} 25/6 \\ 5/2 \\ 1 \end{bmatrix} t = \begin{bmatrix} 25 \\ 15 \\ 6 \end{bmatrix} t$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} e^{3t} + c_3 \begin{bmatrix} 25 \\ 15 \\ 6 \end{bmatrix} e^{5t}}}$$

← Liker denne best.

Hvilken ber oppgava om?

$$\underline{\underline{= \begin{bmatrix} 1 & 3 & 25 \\ 0 & 1 & 15 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} c_1 e^{2t} \\ c_2 e^{3t} \\ c_3 e^{5t} \end{bmatrix}}}$$

b) $A = \begin{bmatrix} 3 & -1 & 2 \\ 3 & -1 & 6 \\ -2 & 2 & -2 \end{bmatrix}$

$\det(A - \lambda I_3) = 0$

$$(3-\lambda)\begin{vmatrix} -1-\lambda & 6 \\ 2 & -2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3 & 6 \\ -2 & -2-\lambda \end{vmatrix} + 2\begin{vmatrix} 3 & -1-\lambda \\ -2 & 2 \end{vmatrix} = 0$$

$$(3-\lambda)(-1-\lambda)(-2-\lambda) - 12(3-\lambda) + (-6-3\lambda+12) + 2(6-2-2\lambda) = 0$$

$$-\lambda^3 + 7\lambda + 6 - 36 + 12\lambda - 6 - 3\lambda + 12 + 12 - 4 - 4\lambda = 0$$

$$\underline{-\lambda^3 + 12\lambda - 16 = 0}$$

Ved testing: $\lambda = 2$ løser:

$$-\lambda^3 + 12\lambda - 16 : (\lambda - 2) = -\lambda^2 - 2\lambda + 8 : -(\lambda-2)(\lambda+4)$$

$$\begin{array}{l} \underline{-\lambda^3 + 2\lambda^2} \\ \quad -2\lambda^2 + 12\lambda \\ \quad \underline{-2\lambda^2 + 4\lambda} \\ \qquad\quad 8\lambda - 16 \\ \qquad\quad \underline{8\lambda - 16} \\ \qquad\qquad 0 \end{array}$$

$\Rightarrow \lambda_1 = 2,\ \lambda_2 = 2$ og $\lambda_3 = -4$

$\underline{\lambda_1 = \lambda_2 = 2}$:

$$A - 2I_3 = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -2 & 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{V_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot t,\ \underline{V_2} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \cdot t$$

$\underline{\lambda_3 = -4}$:

$$A + 4I_3 = \begin{bmatrix} 7 & -1 & 2 \\ 3 & 3 & 6 \\ -2 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 2 \\ 7 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 3 \\ 0 & 6 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \underline{V_3} = \begin{bmatrix} -1/2 \\ -3/2 \\ 1 \end{bmatrix} \cdot t = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \cdot t$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{2t} + c_3 \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} e^{-4t}}}$$

R

Oppgave 3:

a) A er lik oppgave 2a)

$$\Rightarrow y(t) = c_1 \begin{bmatrix}1\\0\\0\end{bmatrix} e^{2t} + c_2 \begin{bmatrix}3\\1\\0\end{bmatrix} e^{3t} + c_3 \begin{bmatrix}25\\15\\6\end{bmatrix} e^{5t}$$

$$y(0) = \begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 3 & 25\\0 & 1 & 15\\0 & 0 & 6\end{bmatrix} \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}c_1 + 3c_2 + 25c_3\\c_2 + 15c_3\\6c_3\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow \underline{c_1 = 1,\ c_2 = 0 = c_3}$$

$$\Rightarrow \underline{\underline{y(t) = \begin{bmatrix}1\\0\\0\end{bmatrix} e^{2t}}}$$

R

b) A er lik oppgave 2b)

$$\Rightarrow y(t) = c_1 \begin{bmatrix}1\\1\\0\end{bmatrix} e^{2t} + c_2 \begin{bmatrix}-2\\0\\1\end{bmatrix} e^{2t} + c_3 \begin{bmatrix}1\\3\\-2\end{bmatrix} e^{-4t} \qquad y(0) = \begin{bmatrix}1\\-1\\1\end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c}1 & -2 & 1 & 1\\1 & 0 & 3 & -1\\0 & 1 & -2 & 1\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & -2 & 1 & 1\\0 & 2 & 2 & -2\\0 & 1 & -2 & 1\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & -2 & 1 & 1\\0 & 1 & 1 & -1\\0 & 0 & -3 & 2\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c}1 & -2 & 0 & 5/3\\0 & 1 & 0 & -1/3\\0 & 0 & 1 & -2/3\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & 0 & 0 & 1\\0 & 1 & 0 & -1/3\\0 & 0 & 1 & -2/3\end{array}\right]$$

$$\Rightarrow \underline{\underline{y(t) = \begin{bmatrix}1\\1\\0\end{bmatrix} e^{2t} - \frac{1}{3} \begin{bmatrix}-2\\0\\1\end{bmatrix} e^{2t} - \frac{2}{3} \begin{bmatrix}1\\3\\-2\end{bmatrix} e^{-4t}}}$$

R

Oppgave 4: ~~2~~ b)

$$\Rightarrow y(t) = c_1\begin{bmatrix}1\\1\\0\end{bmatrix}e^{2t} + c_2\begin{bmatrix}-2\\0\\1\end{bmatrix}e^{2t} + c_3\begin{bmatrix}1\\3\\-2\end{bmatrix}e^{-4t}$$

$$y(0) = \underbrace{\underline{y_0}}_{\underline{b}} = c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}-2\\0\\1\end{bmatrix} + c_3\begin{bmatrix}1\\3\\-2\end{bmatrix} = \underbrace{\begin{bmatrix}1 & -2 & 1\\1 & 0 & 3\\0 & 1 & -2\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}}_{\underline{x}}$$

$\Rightarrow$ Dersom $A\underline{x} = \underline{b}$ er løsbar, vil systemet ha en entydig løsning for initialverdien $y(0) = \underline{y_0}$

Kan løses dersom A er inverterbar
A er inverterbar dersom $\det(A) \neq 0$

$$\det A = 1\begin{vmatrix}0 & 3\\1 & -2\end{vmatrix} - 1\begin{vmatrix}-2 & 1\\1 & -2\end{vmatrix} + 0 = -3-(4+1) = -6 \neq 0$$

R

$\Rightarrow A\underline{x} = \underline{b}$ er løsbar s.a. $\underline{x} = \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}$ er en entydig løsning $\blacksquare$

Oppgave 5: $\begin{bmatrix}x(t)\\y(t)\end{bmatrix}' = \begin{bmatrix}0 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x(t)\\y(t)\end{bmatrix} + \frac{1}{2}\begin{bmatrix}3e^{-2t}\\e^{-2t}\end{bmatrix}$

a) likning på formen $y' = Ay$, allerede løst flere slike.

$$\det(A - I_2\lambda) = \begin{bmatrix}-\lambda & 1\\2 & 1-\lambda\end{bmatrix} = \lambda^2 + \lambda - 2 = 0$$
$$(\lambda+1)(\lambda-2) = 0$$
$$\underline{\lambda_1 = -1\ ,\ \lambda_2 = 2}$$

$\underline{\lambda_1 = -1:}$
$A + I_2 = \begin{bmatrix}1 & 1\\2 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix} \Rightarrow \underline{V_1} = \begin{bmatrix}-1\\1\end{bmatrix}\cdot t$

$\underline{\lambda_2 = 2:}$
$A - 2I_2 = \begin{bmatrix}-2 & 1\\2 & -1\end{bmatrix} \sim \begin{bmatrix}2 & -1\\0 & 0\end{bmatrix} \Rightarrow \underline{V_2} = \begin{bmatrix}1\\2\end{bmatrix}\cdot t$

$\Rightarrow$ Egenrommet ligger i

$\mathrm{sp}\left\{\begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\right\}$

b) $y = \begin{bmatrix} 0 \\ \frac{1}{2}e^{-2t} \end{bmatrix}$ løsning?

V.S. $y' = \begin{bmatrix} 0 \\ -e^{-2t} \end{bmatrix}$

H.S.: $\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} y + \frac{1}{2} \begin{bmatrix} 3e^{-2t} \\ e^{-2t} \end{bmatrix} =$

$$\begin{bmatrix} \frac{1}{2}e^{-2t} \\ \frac{1}{2}e^{-2t} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3e^{-2t} \\ e^{-2t} \end{bmatrix} = \begin{bmatrix} 2e^{-2t} \\ e^{-2t} \end{bmatrix}$$

V.S $\neq$ H.S. ikke en løsning

R

$y = \frac{1}{2}\begin{bmatrix} -2e^{-2t} \\ e^{-2t} \end{bmatrix}$ løsning?

V.S: $y' = \begin{bmatrix} 2e^{-2t} \\ -e^{-2t} \end{bmatrix}$

H.S.: $\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} y + \frac{1}{2} \begin{bmatrix} 3e^{-2t} \\ e^{-2t} \end{bmatrix} =$

$$\begin{bmatrix} \frac{1}{2}e^{-2t} \\ -2e^{-2t} + \frac{1}{2}e^{-2t} \end{bmatrix} + \begin{bmatrix} \frac{3}{2}e^{-2t} \\ \frac{1}{2}e^{-2t} \end{bmatrix} = \begin{bmatrix} 2e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

V.S. = H.S. $\Rightarrow$ $y = \frac{1}{2}\begin{bmatrix} -2e^{-2t} \\ e^{-2t} \end{bmatrix}$

R

c) Bruker teorem 13.24 og resultater fra a) og b)

$$y(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + \frac{1}{2}\begin{bmatrix} -2e^{-2t} \\ e^{-2t} \end{bmatrix}$$

R

Kapittel 14

Oppgave 1 Disse er homogene:

a) $y''-y=0$ la $v=y'$

$y''=y$

$$\Rightarrow \begin{bmatrix} y \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

R

b) $y''+2y'+3y=0$ la $v_1=y$ og $v_2=y'$

$y''=-2y'-3y$

$v_1'=v_2$

$v_2'=y''$

$$\Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

R

c) $y''+y'=0$ la $v=y'$ $\Rightarrow$ $v'=y''$

$y''=-y'$

$$\Rightarrow \begin{bmatrix} y \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} y \\ v \end{bmatrix}$$

R

Oppgave 2: homogene:

a) $y''-y'-2y=0$ $v_1=y$ og $v_2=y'$ $\Rightarrow$ $v_1'=v_2$

$v_2'=y''=y'-2y$

$\underline{y''=y'+2y}$

$$\Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

λ: $\det\begin{pmatrix} -\lambda & 1 \\ 2 & 1-\lambda \end{pmatrix} = \lambda^2+\lambda-2=0 \Rightarrow (\lambda+1)(\lambda-2)=0 \Rightarrow \underline{\lambda=-1 \lor \lambda=2}$

$\underline{\lambda = -1:}$

$$A + I_2 = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \cdot t$$

$\underline{\lambda = 2:}$

$$A - 2I_2 = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot t$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}}} \Rightarrow \underline{\underline{y(t) = c_1 \cdot e^{-t} + c_2 e^{2t}}}$$ R

b) $y'' + y = 0$

$y'' = -y$

La $V = y'$ $\Rightarrow V' = y'' = -y$

$y' = V$

$$\begin{bmatrix} y \\ V \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y \\ V \end{bmatrix}$$

$$\det(A - \lambda I_2) = -\lambda^2 + 1 \Rightarrow \lambda^2 = -1 \Rightarrow \underline{\lambda_1 = i} \; (\lambda_2 = -i)$$

$$A - iI_2 = \begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \sim \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{V_1} = \underline{\begin{bmatrix} -i \\ 1 \end{bmatrix} \cdot t}$$

Teorem 13.16: $\alpha = 0$, $\beta = 1$, $\underline{V_2} = \overline{V_1}$

$$\begin{bmatrix} y \\ V \end{bmatrix} = c_1\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cos t - \begin{bmatrix} -1 \\ 0 \end{bmatrix} \sin t\right) + \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin t + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \cos t\right) c_2$$

$$= c_1 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} + c_2 \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix} = \begin{bmatrix} \sin t & -\cos t \\ \cos t & \sin t \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \sin t + c_2 \cos t}}$$ R

Flippa fortegnet på konstantleddet har ikke så mye å si for generell løsning

c) $y'' - 4y' + 4y = 0$

$y'' = 4y' - 4y$ La $\underline{v_1 = y,\ v_2 = y'}$ => $\underline{v_2 = v_1'}$

$v_2' = y'' = 4y' - 4y$
$\underline{= 4v_2 - 4v_1}$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\lambda:\ \det(A - \lambda I_2) = \begin{vmatrix} -\lambda & 1 \\ -4 & 4-\lambda \end{vmatrix} = -4\lambda + \lambda^2 + 4 = 0$$

$$(\lambda - 2)^2 = 0$$

$$\underline{\lambda_1 = \lambda_2 = 2}$$

$$\Rightarrow (ye^{-2t})'' = (y'e^{-2t} + 2ye^{-2t})' = y''e^{-2t} - 2y'e^{-2t} - 2y'e^{-2t} + 4ye^{-2t}$$

$$= e^{-2t}(y'' - 4y' + 4y)$$

(Eks 14.3)

Ved dobbel integresjon: $\underline{\underline{y(t) = e^{2t}(c_1 t + c_0)}}$ R

Skjønner ikke helt hvordan denne løses... (selv om jeg fikk det til)

Oppgave 3: Samme likninger som i 2

a) $y(0) = 0 \Rightarrow c_1 + c_2 = 0 \Rightarrow c_1 = -c_2$
$y'(0) = 1 \Rightarrow \cancel{7}c_1 + 2c_2 = 1 \Rightarrow c_1 = -(1 - 2c_2)$

$\Big\}\ c_2 = 1 - 2c_2 \Rightarrow c_2 = \frac{1}{3}$
$\Rightarrow c_1 = -\frac{1}{3}$

$$\Rightarrow \underline{\underline{y(t) = \tfrac{1}{3}(e^{2t} - e^{-t})}}$$ R

b) $y(\frac{\pi}{2})=1 \Rightarrow c_1 = 1$

$y'(\frac{\pi}{2}) = 0 \Rightarrow c_2 = 0$

$\Rightarrow \underline{\underline{y(t) = \sin t}}$ R

c) $y(1)=0 \Rightarrow e^2(c_1+c_0)=0 \Rightarrow \underline{c_1 = -c_0}$

$$y' = 2e^{2t}(c_1 t + c_0) + e^{2t}c_1$$
$$= e^{2t}(2c_1 t + 2c_0 + c_1)$$

$y'(0) = -e^{-2} \Rightarrow \underline{2c_0 + c_1 = -e^{-2t}}$

(I) $c_1 = -c_0$
(II) $2c_0 + c_1 = -e^{2t}$

I inn i II: $\underline{c_0 = -e^{-2}}$

$\Rightarrow c_1 = e^{-2}$

$\Rightarrow y(t) = e^{2t}(e^{-2}t - e^{-2}) = e^{2t-2}(t-1) = \underline{e^{2(t-1)}(t-1)}$

$\underline{\underline{y(t) = e^{2(t-1)}(t-1)}}$ R

~~Oppgave 4: a) homogen løsning fra 2a) $y_h(t) = c_1 e^{-t} + c_2 e^{2t}$~~

$\cancel{\Rightarrow y_1 = e^{-t}, \ y_2 = e^{2t}}$

$$\cancel{y_p(t) = e^{2t}\int_0^t \frac{e^{-s}e^{-2s}}{e^{-s}2e^{2s} + e^{2s}e^{-s}}\,ds - e^{-t}\int_0^t \frac{e^{2s}\cdot e^{-2s}}{e^{-s}2e^{2s} + e^{2s}e^{-s}}\,ds = e^{2t}\int_0^t \frac{e^{-3s}}{2e^{s}}\,ds - e^{-t}\int_0^t \frac{1}{2e^{s}}\,ds}$$

$$\cancel{= \frac{e^{2t}}{2}\int_0^t e^{-4s}\,ds - \frac{e^{-t}}{2}\int_0^t e^{-s}\,ds}$$

$$\cancel{= \frac{e^{2t}}{2}\left(-\frac{1}{4}(e^{-4t}-1)\right) - \frac{e^{-t}}{2}(-e^{-t}+1)}$$

Oppgave 4: a) fra 2a, homogen: $y_h(t) = \underbrace{c_1 e^{-t}}_{y_1} + \underbrace{c_2 e^{2t}}_{y_2}$

$f(t) = e^{-2t}$

$y_1 \cdot f(t) = e^{-3t}$
$y_2 \cdot f(t) = e^0 = 1$

$y_1 \cdot y_2' - y_2 \cdot y_1' = e^{-t} \cdot 2e^{2t} + e^{-t} \cdot e^{2t} = \underline{3e^t}$

$$y_p = e^{2t} \int \frac{e^{-3t}}{3e^t}\,dt - e^{-t} \int \frac{1}{3e^t}\,dt = \frac{e^{2t}}{3} \cdot \left[-\frac{1}{4}e^{-4t}\right] - \frac{e^{-t}}{3} \cdot \left[-e^{-t}\right]$$

$$= -\frac{e^{-2t}}{12} + \frac{e^{-2t}}{3} = \underline{\frac{e^{-2t}}{4}}$$

$$\Rightarrow \underline{\underline{y(t) = c_1 e^{-t} + c_2 e^{2t} + \frac{1}{4}e^{-2t}}}$$

R

b) Nesten samme stykke som a); $y_h(t)$ er lik, y_1 og y_2 er og like

$$\left.\begin{array}{l} f(t) = e^{2t} \\ y_1 \cdot f(t) = e^{t} \\ y_2 \cdot f(t) = e^{4t} \end{array}\right\}$$

$$y_p = e^{2t} \int \frac{e^{t}}{3e^t}\,dt - e^{-t} \int \frac{e^{4t}}{3e^t}\,dt$$

$$= \frac{e^{2t}}{3}\left[t\right] - \frac{e^{-t}}{3}\left[\frac{1}{3}e^{3t}\right]$$

$$y_p = e^{2t} \cdot \frac{1}{3}\left(t - \frac{1}{3}\right)$$

"absorberes" inn i c_2

$$\Rightarrow \underline{\underline{y(t) = c_1 e^{-t} + c_2 e^{2t} + \frac{1}{3}t e^{2t}}}$$

R

c) homogen løsning fra 2b: $y_h(t) = c_1 \underbrace{\sin t}_{y_2} + c_2 \underbrace{\cos t}_{y_1}$

- $f(t) = t$
$y_2 \cdot f(t) = t \sin t$
$y_1 \cdot f(t) = t \cos t$

$y_1 \cdot y_2' - y_1' \cdot y_2 = \cos^2 t + \sin^2 t = 1$

$$y_p = \sin t \int \frac{t \cos t}{1} dt - \cos t \int \frac{t \sin t}{1} dt$$

$$= \sin t \left[t \sin(t) + \cos(t)\right] - \cos t \left[-t \cos t + \sin t\right]$$

$$= t \cdot \sin^2 t + \cancel{\sin(t) \cos(t)} + t \cdot \cos^2 t - \cancel{\sin t \cdot \cos t}$$

$$= t (\sin^2 t + \cos^2 t) = \underline{t}$$

$$\Rightarrow \underline{\underline{y(t) = c_1 \sin t + c_2 \cos t + t}}$$

R

d) homogen løsning fra 2c): $y_h(t) = e^{2t}(c_1 t + c_0)$
$= c_1 \underbrace{e^{2t} \cdot t}_{y_1} + \underbrace{e^{2t}}_{y_2} c_0$

$f(t) = 4$
$y_1 \cdot f(t) = 4t^2 e^{2t}$
$y_2 \cdot f(t) = 4t e^{2t}$

$y_1 \cdot y_2' - y_1' \cdot y_2 = \cancel{2te^{4t}} - e^{2t}(e^{2t} + \cancel{2te^{4t}}) = \underline{-e^{4t}}$

$\Rightarrow y_p = e^{2t} \int \frac{4t^2 e^{2t}}{-e^{4t}} dt - e^{2t} \int \frac{4te^{2t}}{-e^{4t}} dt$

$= -4e^{2t} \int \frac{t^2}{e^{2t}} - \frac{t}{e^{2t}} dt = -4e^{2t} \left[\frac{1}{4}(-2e^{-2t} \cdot t - e^{-2t}) - \frac{1}{4}(-2e^{-2t} t - e^{-2t})\right]$

$= -e^{2t} \left[e^{-2t}(-t - 1 + 2t\right.$

d) homogen løsning fra 2c): $(c_1 + tc_2)e^{2t} = \underbrace{e^{2t}}_{y_1} c_1 + \underbrace{te^{2t}}_{y_2} c_2$

$f(t) = 4t$

$y_1 \cdot f(t) = 4t \cdot e^{2t}$
$y_2 \cdot f(t) = 4t^2 e^{2t}$

$y_1 \cdot y_2' - y_1' \cdot y_2 = e^{2t} \cdot (e^{2t} + 2te^{2t}) - 2e^{2t} \cdot te^{2t} = e^{4t}$

$\Rightarrow y_p = e^{2t} \left(t \cdot \int \frac{4t}{e^{2t}} dt - \int \frac{4t^2}{e^{2t}} dt \right)$

$= \cancel{e^{2t}} \cdot t \left[-2\cancel{e^{-2t}} t - \cancel{e^{-2t}} \right] - \cancel{e^{2t}} \left[-2\cancel{e^{-2t}} t^2 - 2\cancel{e^{-2t}} \cdot t - \cancel{e^{-2t}} \right]$

$= \cancel{-2t^2} \cancel{-t} + \cancel{2t^2} + \cancel{2}t + 1 = \underline{t+1}$

$\Rightarrow \underline{\underline{y(t) = (c_1 + tc_2)e^{2t} + t + 1}}$ R

Oppgave 5: (a) polynom: $\lambda^2 - 1 = 0$
matrise: $(-\lambda)^2 - 1 = 0 \Rightarrow \lambda^2 - 1 = 0$

(b) polynom: $\lambda^2 + 2\lambda + 3 = 0$
matrise: $\lambda^2 + 2\lambda + 3 = 0$

(c) polynom: $\lambda^2 + \lambda = 0$
matrise: $\lambda^2 + \lambda = 0$

$\Rightarrow$ De er like

P = Polynom: $y''(t) + a_1 y'(t) + a_0 y(t)$
Karakteristisk likning, P: $\underline{\lambda^2 + a_1\lambda + a_0 = 0}$

Kan skrives om til system: $\begin{bmatrix} y \\ y' \end{bmatrix}' = \underbrace{\begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}}_{=A} \begin{bmatrix} y \\ y' \end{bmatrix}$

Karakteristisk likning A:
$\det(A - \lambda I_2) = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 \\ -a_0 & -a_1 - \lambda \end{vmatrix} = \underline{\lambda^2 + a_1\lambda + a_0 = 0}$ $\underline{\underline{\text{De blir like}}}$ ■

Flott.

R

God sommer og lykke til på eksamen ☺