Innlevering 4 - Matematikk 3

Oppgave 1: Dersom $\vec{u} \cdot \vec{v} = 0$ er $\vec{u}$ og $\vec{v}$ ortogonale.

a) $\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 2 \cdot 1 + (-5) \cdot 2 + 0 \cdot 1 = -8 \neq 0 \Rightarrow$ ikke ortogonale

R

b) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} = 1 + 0 - 1 = 0 \Rightarrow$ ortogonale

$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} = 1 + 0 - 1 = 0 \Rightarrow$ ortogonale

R

$\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} = 1 - 2 + 1 = 0 \Rightarrow$ ortogonale

Alle vektorene er ortogonale med hverandre

c) $\begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -i & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} = 1 \neq 0 \Rightarrow$ ikke ortogonale R

Må konjugeres

$\begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ i \\ i \end{bmatrix} = \begin{bmatrix} -i & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ i \\ i \end{bmatrix} = -i + i = 0 \Rightarrow$ Ortogonale R

$\begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ i \\ i \end{bmatrix} = \begin{bmatrix} -i & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ i \\ i \end{bmatrix} = -i + i = 0 \Rightarrow$ ortogonale R

$\begin{bmatrix} 1 \\ i \\ i \end{bmatrix}$ er ortogonal med de andre vektorene, men de to andre vektorene er ikke ortogonale med hverandre

R

Oppgave 2: Kaller vektorene i hver oppgave $\vec{v}_n$ fra venstre

a) $\vec{u}_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{\langle \vec{v}_2, \vec{u}_1 \rangle}{\langle \vec{u}_1, \vec{u}_1 \rangle} \vec{u}_1$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{-8}{30} \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 23/15 \\ 2/3 \\ 4/15 \end{bmatrix}$$

Ortogonal basis: $\left(\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 23/15 \\ 2/3 \\ 4/15 \end{bmatrix} \right)$ R

b) $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} - \frac{\langle \vec{u}_1, \vec{v}_2 \rangle}{\langle \vec{u}_1, \vec{u}_1 \rangle} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} - 0 \cdot \vec{u}_1 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$

$\vec{u}_3 = \vec{v}_3$ (vektorene er allt ortogonale)

Ortogonal basis: $\left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} \right)$ R

c) Velger $\vec{u}_1 = \begin{bmatrix} 1 \\ i \\ i \end{bmatrix}$, siden den er ortogonal med begge de andre

$$\Rightarrow \vec{u}_2 = \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_3 = \begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix} - \frac{\langle \vec{v}_1, \vec{u}_2 \rangle}{\langle \vec{u}_2, \vec{u}_2 \rangle} \vec{u}_2 = \begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} i/2 \\ -1/2 \\ 1 \end{bmatrix}$$

skipper $P_{\vec{u}_1}$, $\vec{u}_1 \perp \vec{v}_1$)

Ortogonal basis: $\left(\begin{bmatrix} 1 \\ i \\ i \end{bmatrix}, \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} i/2 \\ -1/2 \\ 1 \end{bmatrix} \right)$ R

Oppgave 3: Kaller vektorene i hver deloppgave $\vec{v}_n$ fra venstre → $\vec{w}$, vektoren som skal projiseres.

a) $P_{\vec{v}_1}(\vec{w}) = \frac{\langle \vec{v}_1, \vec{w} \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} \cdot \vec{v}_1 = -\frac{5}{30}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -1/3 \\ 5/6 \\ -1/6 \end{bmatrix}}}$

$P_{\vec{v}_2}(\vec{w}) = \frac{\langle \vec{v}_2, \vec{w} \rangle}{\langle \vec{v}_2, \vec{v}_2 \rangle} \cdot \vec{v}_2 = \frac{5}{5}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}}$

Må bruke den ortogonale basisen her, samt plusse de to projeksjonene sammen.

b) $P_{\vec{v}_1}(\vec{w}) = \frac{\langle \vec{v}_1, \vec{w} \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} \cdot \vec{v}_1 = \underline{\underline{\vec{0}}}$

$P_{\vec{v}_2}(\vec{w}) = \frac{\langle \vec{v}_2, \vec{w} \rangle}{\langle \vec{v}_2, \vec{v}_2 \rangle} \cdot \vec{v}_2 = \frac{6}{4}\begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 3/2 \\ 3\sqrt{2}/2 \\ 3/2 \end{bmatrix}}}$

$P_{\vec{v}_3}(\vec{w}) = \frac{\langle \vec{v}_3, \vec{w} \rangle}{\langle \vec{v}_3, \vec{v}_3 \rangle} \cdot \vec{v}_3 = \frac{2}{4}\begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1/2 \\ -\sqrt{2}/2 \\ 1/2 \end{bmatrix}}}$

Plusse sammen her også. Kanskje du ser noe spennende?

c) $P_{\vec{v}_1}(\vec{w}) = \frac{\langle \vec{v}_1, \vec{w} \rangle}{\langle \vec{v}_1, \vec{v}_1 \rangle} \cdot \vec{v}_1 = \frac{1+i}{2}\begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{i-1}{2} \\ 0 \\ \frac{i+1}{2} \end{bmatrix}}}$

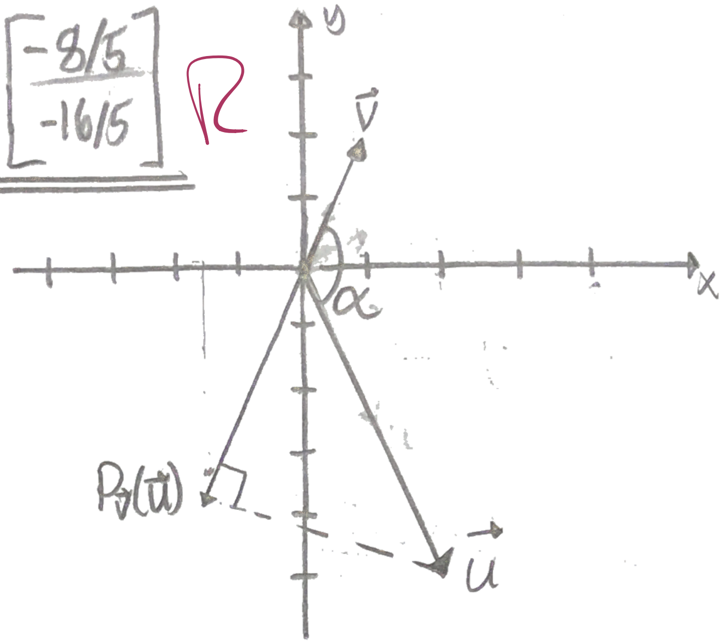
$P_{\vec{v}_2}(\vec{w}) = \frac{\langle \vec{v}_2, \vec{w} \rangle}{\langle \vec{v}_2, \vec{v}_2 \rangle} \vec{v}_2 = \frac{1+i}{2}\begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{i-1}{2} \\ \frac{i+1}{2} \\ 0 \end{bmatrix}}}$

$P_{\vec{v}_3}(\vec{w}) = \frac{\langle \vec{v}_3, \vec{w} \rangle}{\langle \vec{v}_3, \vec{v}_3 \rangle} \vec{v}_3 = \frac{i+2}{3}\begin{bmatrix} 1 \\ i \\ i \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{i+2}{3} \\ \frac{2i-1}{3} \\ \frac{2i-1}{3} \end{bmatrix}}}$

Samme som over.

Oppgave 4:

$$P_{\vec v}(\vec u) = \frac{\langle \vec v, \vec u \rangle}{\langle \vec v, \vec v \rangle}\vec v = \frac{-8}{5} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -8/5 \\ -16/5 \end{bmatrix}}}$$ R



$$\cos\alpha = \frac{\vec v \cdot \vec u}{|\vec v||\vec u|} = \frac{-8}{\sqrt5 \cdot \sqrt{29}}$$

$$\alpha = \cos^{-1}\left(\frac{-8}{\sqrt{145}}\right)$$ R

$$\underline{\underline{\alpha = 131{,}6^\circ}}$$

Oppgave 5

a) $$P_{\vec u}(\vec v) = \frac{\langle \vec u, \vec v \rangle}{\langle \vec u, \vec u \rangle}\vec u = \frac{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \frac{11}{9}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 22/9 \\ 22/9 \\ 11/9 \end{bmatrix}}}$$ R

b) $$P_{\vec u}(\vec v) = \frac{\langle \vec u, \vec v \rangle}{\langle \vec u, \vec u \rangle}\vec u = \frac{\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}}{\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \frac{4}{6}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -2/3 \\ 4/3 \\ -2/3 \end{bmatrix}}}$$ R

c) $$P_{\vec u}(\vec v) = \frac{\langle \vec u, \vec v \rangle}{\langle \vec u, \vec u \rangle}\vec u = \frac{\begin{bmatrix} -i & -i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} -i & -i & 1 \end{bmatrix}\begin{bmatrix} i \\ i \\ 1 \end{bmatrix}} \begin{bmatrix} i \\ i \\ 1 \end{bmatrix} = \frac{1-2i}{3}\begin{bmatrix} i \\ i \\ 1 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} \frac{2+i}{3} \\ \frac{2+i}{3} \\ \frac{1-2i}{3} \end{bmatrix}}}$$ R

Oppdaget at jeg har oversett $\vec v - P_u(\vec v)$, de kommer på neste side

a) $\vec{v} - P_{\vec{u}}(\vec{v}) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 22/9 \\ 22/9 \\ 11/9 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5/9 \\ -4/9 \\ -2/9 \end{bmatrix}}}$ R

b) $\vec{v} - P_{\vec{u}}(\vec{v}) = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} -2/3 \\ 4/3 \\ -2/3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix}}}$ R

c) $\vec{v} - P_{\vec{u}}(\vec{v}) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{2+i}{3} \\ \frac{2+i}{3} \\ \frac{1-2i}{3} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1-i}{3} \\ \frac{1-i}{3} \\ \frac{2+2i}{3} \end{bmatrix}}}$ R

Bra!

d) <u>Det er ortogonale vektorer, dermed blir produktet 0</u> R

<u>Oppgave 6:</u> La $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \vec{w}$

a) $P_{\vec{v}}(\vec{w}) = \frac{\langle \vec{v}, \vec{w} \rangle}{\langle \vec{v}, \vec{v} \rangle} \cdot \vec{v} = \frac{1}{3} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}}}$

b) $[P_{\vec{v}}] = [P_{\vec{v}}(\vec{e}_1) \mid P_{\vec{v}}(\vec{e}_2) \mid P_{\vec{v}}(\vec{e}_3)]$ klarer å se svarene ut ifra a)

$$\underline{\underline{[P_{\vec{v}}] = \begin{bmatrix} 1/3 & -1/3 & 1/3 \\ -1/3 & 1/3 & -1/3 \\ 1/3 & -1/3 & 1/3 \end{bmatrix}}}$$ R

c) $[P_{\vec{v}}]$ danner en linje. Med uendelig antall vektorer: Flere vektorer i $\mathbb{R}^3$ kan gi samme utverdi for $[P_{\vec{v}}]$
=> <u>ikke injektiv</u>
Alle vektorer langs linja ligger i $\mathbb{R}^3$ => <u>surjektiv</u>

Men det spenner ikke ut $\mathbb{R}^3$ => Ikke <u>surjektiv.</u>

d):

$\text{Col}\,[P_{\vec{v}}] = \text{sp}\left\{\begin{bmatrix}1\\-1\\1\end{bmatrix}\right\}$: rommet utspent av matrisen

$\text{Null}\,[P_{\vec{v}}] \perp \text{Col}\,[P_{\vec{v}}] \Rightarrow \text{sp}\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\end{bmatrix}\right\} = \text{Null}\,[P_{\vec{v}}]$

$\begin{bmatrix}1\\-1\\1\end{bmatrix} \cdot \begin{bmatrix}a\\b\\c\end{bmatrix}$

$\rightarrow \quad 1 \cdot a - b + c = 0 \qquad 1a = a = b = 1 \;,\; c = 0$

tilsvarende for den andre, men passer på lineær uavh.

$$\text{ker}\,[P_{\vec{v}}] = \text{Null}\,[P_{\vec{v}}] = \text{sp}\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\end{bmatrix}\right\}$$

$$\text{Col}\,[P_{\vec{v}}] = \text{Im}\,[P_{\vec{v}}] = \text{sp}\left\{\begin{bmatrix}1\\-1\\1\end{bmatrix}\right\}$$

R

Oppgave 7:

a) Velger $\vec{w}_1 = \begin{bmatrix}2\\-5\\1\end{bmatrix}$

$$\vec{w}_2 = \begin{bmatrix}4\\-4\\2\end{bmatrix} - \frac{\langle \vec{w}_1, \vec{v} \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle} \vec{w}_1 = \begin{bmatrix}4\\-4\\2\end{bmatrix} - \frac{30}{30}\begin{bmatrix}2\\-5\\1\end{bmatrix}$$

$$\vec{w}_2 = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

$$\text{Basis } W = \text{sp}\left\{\begin{bmatrix}2\\-5\\1\end{bmatrix}, \begin{bmatrix}2\\1\\1\end{bmatrix}\right\}$$

R

b) $P_{\vec{w}_1}(\vec{e}_1) = \dfrac{\langle \vec{w}_1, \vec{e}_1 \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle} \vec{w}_1 = \dfrac{2}{30}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \dfrac{1}{15}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$

Har gjort en del projeksjoner nå, beregner dem direkte heretter

$$P_{\vec{w}_2}(\vec{e}_1) = \frac{2}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \underline{\frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}}$$

$$P_W(\vec{e}_1) = P_{\vec{w}_1}(\vec{e}_1) + P_{\vec{w}_2}(\vec{e}_1) = \underline{\frac{1}{5}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}}$$

$$\left.\begin{aligned} P_{\vec{w}_1}(\vec{e}_2) &= -\frac{1}{6}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \\ P_{\vec{w}_2}(\vec{e}_2) &= \frac{1}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \end{aligned}\right\} \Rightarrow P_W(\vec{e}_2) = -\frac{1}{6}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

$$\left.\begin{aligned} P_{\vec{w}_1}(\vec{e}_3) &= \frac{1}{30}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \\ P_{\vec{w}_2}(\vec{e}_3) &= \frac{1}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \end{aligned}\right\} \Rightarrow P_W(\vec{e}_3) = \frac{1}{30}\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \underline{\frac{1}{5}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}$$

$$[P_W] = \left[P_W(\vec{e}_1) \,\middle|\, P_W(\vec{e}_2) \,\middle|\, P_W(\vec{e}_3) \right] = \underline{\underline{\begin{bmatrix} 4/5 & 0 & 2/5 \\ 0 & 1 & 0 \\ 2/5 & 0 & 1/5 \end{bmatrix}}}$$

R

c) Kolonne 1 og 3 er lineært avhengige,
$[P_W]$ utspenner to vektorer, som igjen utspenner et plan.
Så lenge A ligger i planet får vi ut A fra likningen.
Kan f.eks. velge A s.a. kolonne 1 = $P_W(\vec{e}_1) \cdot 5$
kolonne 2 = $P_W(\vec{e}_1) \cdot 5 \cdot 2$

$$\begin{bmatrix} 4/5 & 0 & 2/5 \\ 0 & 1 & 0 \\ 2/5 & 0 & 1/5 \end{bmatrix} \underbrace{\begin{bmatrix} 4 & 8 \\ 0 & 0 \\ 2 & 4 \end{bmatrix}}_{= A} = \begin{bmatrix} 4 & 8 \\ 0 & 0 \\ 2 & 4 \end{bmatrix}$$

$\Rightarrow$ Ja, det finnes

Oppgave 8:

a) $\langle x, \cos x\rangle = \int_0^1 \overset{u}{x} \overset{v'}{\cos x}\, dx \overset{\text{delvis integrasjon}}{=} \left[x \cdot \sin x\right]_0^1 - \int_0^1 \sin x\, dx$

$= \sin 1 + \cos 1 - \cos 0 = \underline{\sin 1 + \cos 1 - 1}$

$\langle x, x\rangle = \int_0^1 x^2\, dx = \underline{\frac{1}{3}}$

$\langle x, \sin x\rangle = \int_0^1 \overset{u}{x} \cdot \overset{v'}{\sin x}\, dx \overset{\text{delvis int}}{=} \left[-x \cdot \cos x\right]_0^1 + \int_0^1 \cos x\, dx$

$= \underline{-\sin 1 + \cos 1}$

$\langle \cos x, \cos x\rangle = \int_0^1 \cos^2 x\, dx = \frac{1}{2}\int_0^1 1 + \cos(2x)\, dx$

$= \frac{1}{2}\left[x + \frac{1}{2}\sin(2x)\right]_0^1 = \underline{\frac{1}{2} + \frac{1}{4}\sin(2)}$

$\langle \sin x, \sin x\rangle = \int_0^1 \sin^2 x\, dx = \frac{1}{2}\int_0^1 1 - \cos(2x)\, dx$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^1 = \underline{\frac{1}{2} - \frac{1}{4}\sin(2)}$

$\boxed{\alpha : \text{Vinkel}(x, \cos x), \quad \beta : \text{Vinkel}(x \text{ og } \sin x)}$

$\alpha = \arccos\left(\frac{\langle x, \cos x\rangle}{|x| \cdot |\cos x|}\right) = \arccos\left(\frac{\langle x, \cos x\rangle}{\sqrt{\langle x, x\rangle} \cdot \sqrt{\langle \cos x, \cos x\rangle}}\right)$

$= \arccos\left(\frac{\sin 1 + \cos 1 - 1}{\sqrt{\frac{1}{3}} \cdot \sqrt{\frac{1}{2} + \frac{1}{4} \cdot \sin(2)}}\right) = 0{,}684 = \underline{39{,}16^\circ}$

$\beta = \arccos\left(\frac{\sin 1 - \cos 1}{\sqrt{\frac{1}{3}} \cdot \sqrt{\frac{1}{2} - \frac{1}{4}\sin(2)}}\right) = 0{,}046 = \underline{2{,}61^\circ}$

$\underline{\underline{\text{Vinkel}(x, \cos x) = 39{,}16^\circ > \text{Vinkel}(x, \sin x) = 2{,}61^\circ}}$

$\Rightarrow$ <u>Vinkelen mellom x og sin x er minst</u>

JC

b) $|x - \cos x| = \sqrt{\langle x - \cos x, x - \cos x \rangle}$

$= \sqrt{\int (x - \cos x)^2 \, dx} = \sqrt{\int_0^1 x^2 \, dx - 2\int_0^1 x \cdot \cos x \, dx + \int_0^1 \cos^2 x \, dx}$

$= \sqrt{\frac{1}{3} - 2 \sin 1 + 2 \cos 1 + 2 + \frac{1}{2} + \frac{1}{4} \sin(2)}$

Fra a) $= \sqrt{\frac{17}{6} + \frac{1}{4} \sin(2) - 2 \sin 1 + 2 \cos 1}$

$= 0{,}545 \Rightarrow \underline{\underline{|x - \cos x| = 0{,}545}}$

$|x - \sin x| = \sqrt{\langle x - \sin x, x - \sin x \rangle}$

$\langle x - \sin x, x - \sin x \rangle = \int_0^1 (x - \sin x)^2 dx$

$= \int_0^1 x^2 dx - 2\int_0^1 x \cdot \sin x \, dx + \int_0^1 \sin^2 x \, dx = \frac{1}{3} - 2(\sin 1 - \cos 1) + \frac{1}{2} - \frac{1}{4} \sin(2)$

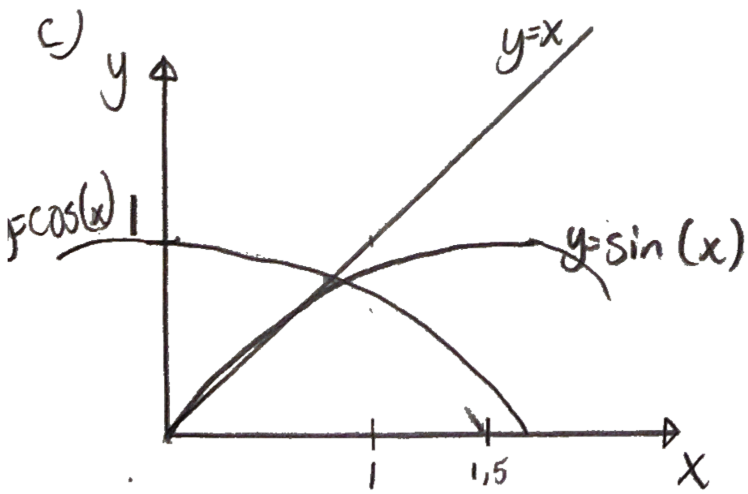
$= \frac{5}{6} + 2 \cos 1 - 2 \sin - \frac{1}{4} \sin(2) = 3{,}67 \cdot 10^{-3}$

$\Rightarrow \underline{\underline{|x - \sin x| = \sqrt{3{,}67 \cdot 10^{-3}} = 0{,}061}}$

<u>Avstanden mellom x og sin x er minst</u>

R

c)



Ser at sin x ligger
tett på x for [0,1]
=> liten vinkel mellom dem,
og liten avstand

R

Oppgave 9

Per def er ortonormal basis slik at $|\vec{u}_j| = 1$ s.a:

$$\langle a_{ij}, a_{ij} \rangle = a_{ij}^T \cdot a_{ij} = 1$$

$$A^T \cdot A = I$$
$$A^T \cdot A \cdot A^{-1} = I \cdot A^{-1}$$
$$A^T \cdot I = A^{-1}$$
$$A^T = A^{-1} \quad \square$$

(Skjønner ikke så mye av dette beviset, fikk mye hjelp av studass.)

9 Antar at $\boldsymbol{u_1}, \ldots, \boldsymbol{u_n}$ er en ortonormal basis for $\mathbb{R}^n$. En invers til A har den egenskapen at $A^{-1}A = I$ og at den er entydig. Så hvis vi klarer å finne en matrise som oppfyller dette så må dette være inversen. La oss se på $A^\top$.

$$A^\top A = \begin{bmatrix} \boldsymbol{u_1} \\ \vdots \\ \boldsymbol{u_n} \end{bmatrix} \begin{bmatrix} \boldsymbol{u_1} & \ldots & \boldsymbol{u_n} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u_1} \cdot \boldsymbol{u_1} & \boldsymbol{u_2} \cdot \boldsymbol{u_1} & \ldots & \boldsymbol{u_n} \cdot \boldsymbol{u_1} \\ \boldsymbol{u_1} \cdot \boldsymbol{u_2} & \boldsymbol{u_2} \cdot \boldsymbol{u_2} & \ldots & \boldsymbol{u_n} \cdot \boldsymbol{u_2} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{u_1} \cdot \boldsymbol{u_n} & \boldsymbol{u_2} \cdot \boldsymbol{u_n} & \ldots & \boldsymbol{u_n} \cdot \boldsymbol{u_n} \end{bmatrix} \tag{68}$$

Men vi vet at $\boldsymbol{u_i} \cdot \boldsymbol{u_j}$ blir 1 hvis $i = j$ og 0 ellers. Dermed får vi at:

$$A^\top A = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & 1 \end{bmatrix} = I \tag{69}$$

Dermed følger det at $A^\top$ må være inversen til A, altså at $A^{-1} = A^\top$.

Ellers veldig bra! Fortsett sånn ☺ Es