Innlevering 1 - Matematikk 1

Kapittel 1

Oppgave 1: Skriver om til polarform, $r = \sqrt{a^2+b^2}$, $a>0 \Rightarrow \theta = \arctan\frac{b}{a}$

$$(\sqrt{3}+i) \Rightarrow r = \sqrt{4} = 2 \ , \ \theta = \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\underline{(\sqrt{3}+i) = 2e^{\frac{\pi}{6}i}}$$

$$(1-i) \Rightarrow r = \sqrt{2}, \ \theta = \arctan\frac{1}{-1} = -\frac{\pi}{4}$$

$$\underline{(1-i) = \sqrt{2}e^{-\frac{\pi}{4}i}}$$

a) $$(\sqrt{3}+i)(1-i) = 2e^{\frac{\pi}{6}i}\cdot\sqrt{2}e^{-\frac{\pi}{4}i} = 2\sqrt{2}e^{(\frac{\pi}{6}-\frac{\pi}{4})i}$$

$$= \underline{\underline{2\sqrt{2}e^{-\frac{\pi}{12}i}}}$$

b) $$\frac{\sqrt{3}+i}{1-i} = \frac{2e^{\frac{\pi}{6}i}}{\sqrt{2}e^{-\frac{\pi}{4}i}} = \sqrt{2}e^{(\frac{\pi}{6}-(-\frac{\pi}{4}))i}$$

$$= \underline{\underline{\sqrt{2}e^{\frac{5\pi}{12}i}}}$$

Flott.

Oppgave 2: Skriver om ved eulers formel:

$$e^z = e^{\frac{3\pi}{4}i} = \cos\left(\frac{3\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right)\cdot i = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

$$e^w = e^{-\frac{3\pi}{4}i} = \cos\left(-\frac{3\pi}{4}\right) + \sin\left(-\frac{3\pi}{4}\right)\cdot i = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

a) $$e^z - e^w = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \sqrt{2}i$$

$$= \underline{\underline{\sqrt{2}e^{\frac{\pi}{2}i}}}$$

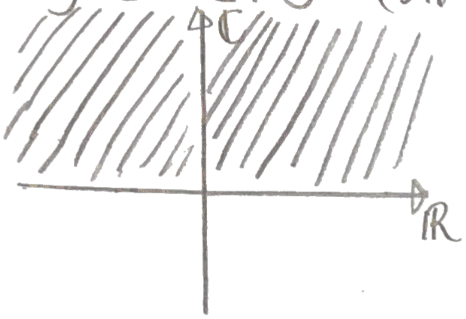
b) $$\frac{e^z}{e^w} = e^{(\frac{3\pi}{4}-(-\frac{3\pi}{4}))i} = e^{\frac{3\pi}{2}i} \ (= -i)$$

$$= \underline{\underline{e^{\frac{3\pi}{2}i}}}$$

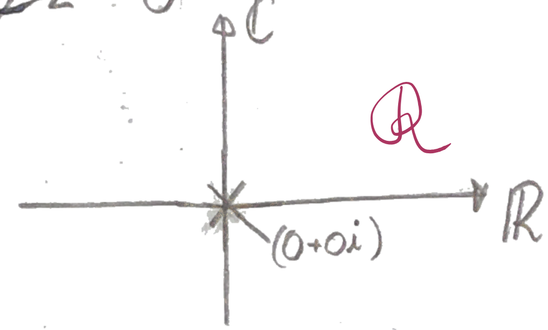
Riktig.

Oppgave 3:

a) $\text{Im } z > 0$ (alt over $\mathbb{R}$-akse)



b) $z^2 = 0$

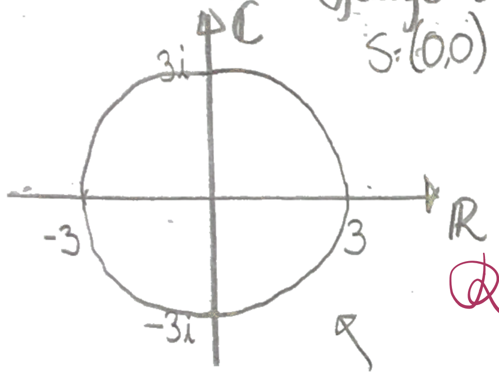


c) $z\bar{z} = 9 \Rightarrow (a+bi)(a-bi) = 9$

$a^2 + b^2 = 9 = 3^2$

Gjenkjenner sirkellikning.

$S:(0,0)$ $r=3$



Alle komplekse tall på buen er løsninger.

d) $z^6 = -1 + i\sqrt{3}$

$z = re^{\theta}$

$\Rightarrow z^6 = r^6 e^{6\theta}$

får $r^6 = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

$r = \pm\sqrt[6]{2}$

$a<b \Rightarrow 6\theta = \arctan\left(\frac{\sqrt{3}}{-1}\right) + \pi + 2\pi \cdot k$

$6\theta = \frac{2\pi}{3} + 2\pi \cdot k$

$k=0 : \Rightarrow \theta = \frac{\pi}{6}$

$k=1 : \Rightarrow \theta = \frac{4\pi}{9}$

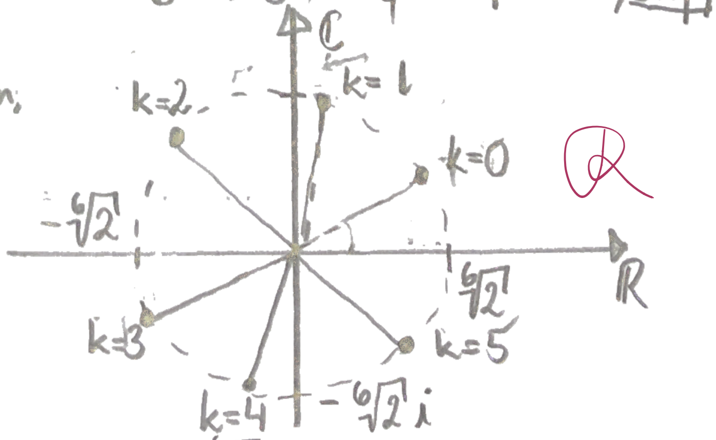
$k=2 \Rightarrow \theta = \frac{7\pi}{9}$

$k=3 \Rightarrow \theta = \frac{10\pi}{9}$

$k=4 \Rightarrow \theta = \frac{13\pi}{9}$

$k=5 \Rightarrow \theta = \frac{16\pi}{9}$

$k=6 \Rightarrow \theta = \frac{19\pi}{9} = \frac{\pi}{9} + 2\pi$, stopp



$r = \pm\sqrt[6]{2}$

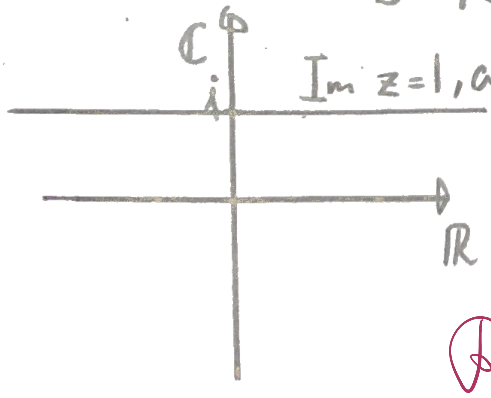
e) $z - (\overline{z - 2i}) = 0$

$a+bi - ((a-bi)-(0+2i)) = 0$

$\cancel{a} + bi - \cancel{a} + bi - 2i = 0$

$2bi = 2i$

$b = 1 \Rightarrow \text{Im } z = 1$



$\text{Im } z = 1$, alle løsninger på denne linjen.

Oppgave 4 Regel

a) La $z = r_1 e^{\theta_1 i} \Rightarrow \bar{z} = r_1 e^{-\theta_1 i}$

La $w = r_2 e^{\theta_2 i} \Rightarrow \bar{w} = r_2 e^{-\theta_2 i}$

Vil bevise $\overline{(z/w)} = \bar{z}/\bar{w}$

Venstre side: $z/w = r_1 e^{\theta_1 i} / r_2 e^{\theta_2 i} = \frac{r_1}{r_2} e^{(\theta_1 - \theta_2) i}$

$$\Rightarrow \overline{z/w} = \frac{r_1}{r_2} e^{(\theta_2 - \theta_1) i}$$

Høyre side: $\bar{z}/\bar{w} = r_1 e^{-\theta_1 i} / r_2 e^{-\theta_2 i} = \frac{r_1}{r_2} e^{(-\theta_1 - (-\theta_2)) i}$

$$\Rightarrow \bar{z}/\bar{w} = \frac{r_1}{r_2} e^{(\theta_2 - \theta_1) i}$$

Høyre side = Venstre side

$$\Rightarrow \overline{(z/w)} = \bar{z}/\bar{w} \quad \text{Q.E.D.}$$

Nydelig!

b) La $z = re^{\theta i} \Rightarrow \bar{z} = re^{-\theta i}$

Vil bevise $(\bar{z})^n = \overline{z^n}$

Venstre side: $(\bar{z})^n = (re^{-\theta i})^n = r^n e^{-n\theta i}$

Høyre side $\overline{z^n} = \overline{(re^{\theta i})^n} = \overline{r^n e^{n\theta i}} = r^n e^{-n\theta i}$

Høyre side = Venstre side $\Rightarrow (\bar{z})^n = \overline{z^n}$, Q.E.D.

Herlig. Bra du bruker $re^{i\theta}$ her!

Oppgave 4:

c) Vil bevise: $|z+w|^2 + |z-w|^2 = 2|z|^2 + 2|w|^2$

Regel: $|z|^2 = z \cdot \bar{z}$	Regel: $\overline{z+w} = \bar{z}+\bar{w}$

Høyre side: $2|z|^2 + 2|w|^2$

Venstre side: $|z+w|^2 + |z-w|^2 = (z+w)(\overline{z+w}) + (z-w)(\overline{z-w}) = *$

$* = (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w}) = **$

$** = z\bar{z} + \cancel{z\bar{w}} + \cancel{\bar{z}w} + w\bar{w} + z\bar{z} - \cancel{z\bar{w}} - \cancel{\bar{z}w} + w\bar{w} = 2z\bar{z} + 2w\bar{w}$

$= 2|z|^2 + 2|w|^2$

Høyre = Venstre $\Rightarrow |z+w|^2 + |z-w|^2 = 2|z|^2 + 2|w|^2$, QED

Perf.

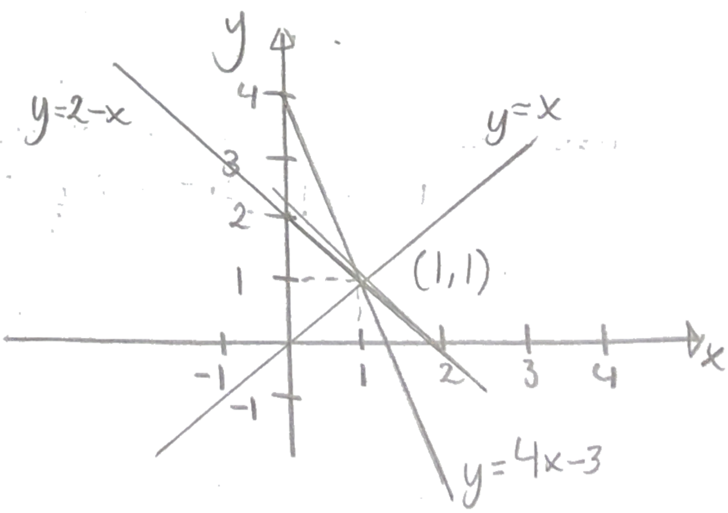
Kapittel 2:

Oppgave 1:

$x + y = 2 \Rightarrow y = 2 - x$

$x - y = 0 \Rightarrow y = x$

$3x + y = 4 \Rightarrow y = 4 - 3x$



Svar: $x=1$, $y=1$

Q

Oppgave 2: Ja, de er ekvivalente. De har de samme løsningene, og de får radekvivalente ~~totalmatriser~~ totalmatriser

$$\begin{cases} x=1 \\ x+y=2 \\ x+y+z=3 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{cases} x=1 \\ y=1 \\ z=1 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

De to matrisene er like, hvilket betyr at totalmatrisene er radekvivalente og:

<u>Likningssystemene er ekvivalente.</u>

R

Oppgave 3:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Nei, de er ikke radekvivalente, de er ikke like like i redusert trappeform $\Rightarrow$ Det er umulig å utføre radoperasjoner på den ene og få den andre.

<u>$\Rightarrow$ De er ikke radekvivalente punkter.</u>

R

Oppgave 4:

a) Nei, det finnes ingen førstegradpolynom som går igjennom 3 punkt som ikke er på linje.

b) Ja. Vi får et likningssett basert på punktene: $f(x)=ax^2+bx+c$

$$\left.\begin{matrix}(-1,3)\\(1,3)\\(2,6)\end{matrix}\right\}\left\{\begin{matrix}a-b+c=3\\a+b+c=3\\4a+2b+c=6\end{matrix}\right\} \Rightarrow \left[\begin{array}{ccc|c}1&-1&1&3\\1&1&1&3\\4&2&1&6\end{array}\right] \Rightarrow *$$

$$* \Rightarrow \left[\begin{array}{ccc|c}1&-1&1&3\\0&2&0&0\\0&2&-3&-6\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&-1&1&3\\0&1&0&0\\0&0&1&2\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&0\\0&0&1&2\end{array}\right]$$

Vi får $f(x)=x^2+2$, den går gjennom alle punktene.

Oppgave 5:

a) Det har én entydig løsning, det er to likninger med to ukjente, likningene er lineært uavhengige av hverandre siden $ad \neq bc$, dermed finnes det én entydig løsning.

b)

$$\left\{\begin{matrix}ax+by=m\\cx+dy=n\end{matrix}\right\} = \left[\begin{array}{cc|c}a&b&m\\c&d&n\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&\frac{b}{a}&\frac{m}{a}\\1&\frac{d}{c}&\frac{n}{c}\end{array}\right] \Rightarrow *$$

$$* \Rightarrow \left[\begin{array}{cc|c}1&\frac{b}{a}&\frac{m}{a}\\0&\frac{d}{c}-\frac{b}{a}&\frac{n}{c}-\frac{m}{a}\end{array}\right] \xRightarrow[\left(\frac{d}{c}-\frac{b}{a}\right)]{} \left[\begin{array}{cc|c}1&\frac{b}{a}&\frac{m}{a}\\0&1&\frac{an-cm}{ad-bc}\end{array}\right] \Rightarrow \left[\begin{array}{cc|c}1&0&\frac{m}{a}-\frac{b}{a}\left(\frac{an-cm}{ad-bc}\right)\\0&1&\frac{an-cm}{ad-bc}\end{array}\right]$$

$$\Rightarrow x = \frac{m}{a} - \frac{b}{a}\left(\frac{an-cm}{ad-bc}\right), \quad y = \frac{an-cm}{ad-bc}$$

Gyldig ettersom $ad \neq bc \Rightarrow$ ikke 0 i nevner

Kunne blitt forenklet algebraisk men who cares.

Oppgave 6:

a)
$$\left.\begin{aligned} 2x-4y+9z&=-38\\ 4x-3y+8z&=-26\\ -2x+4y-2z&=17\end{aligned}\right\}\left[\begin{array}{ccc|c}2&-4&9&-38\\4&-3&8&-26\\-2&4&-2&17\end{array}\right]$$

$$\Rightarrow\left[\begin{array}{ccc|c}2&-4&9&-38\\0&5&-10&50\\0&0&7&-21\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}2&-4&9&-38\\0&1&0&4\\0&0&1&-3\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}1&0&0&5/2\\0&1&0&4\\0&0&1&-3\end{array}\right]$$

$$\Rightarrow x=5/2,\ y=4,\ z=-3$$

R

b)
$$\left[\begin{array}{ccc|c}1&3&6&4\\2&8&16&8\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}1&3&16&4\\1&4&8&4\end{array}\right]$$

Glemte å skrive likningen først

$$\Rightarrow\left[\begin{array}{ccc|c}1&3&6&4\\0&1&2&0\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}1&0&0&4\\0&1&2&0\end{array}\right]\Rightarrow x=4$$

$y+2z=0$

$y=-2t$

La $z=t$:

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\-2t\\t\end{bmatrix}=\begin{bmatrix}4\\0\\0\end{bmatrix}+\begin{bmatrix}0\\-2\\1\end{bmatrix}t$$

R

c)
$$\left.\begin{cases}(1+i)z-w=i\\(1-i)z+(1+i)w=1\end{cases}\right\}\left[\begin{array}{cc|c}1+i&-1&i\\1-i&1+i&1\end{array}\right]$$

$$\begin{array}{c}\\ \text{II}\cdot(i)\end{array}\Rightarrow\left[\begin{array}{cc|c}1+i&-1&i\\1+i&i-1&i\end{array}\right]\Rightarrow\left[\begin{array}{cc|c}1+i&-1&i\\0&i&0\end{array}\right]\Rightarrow\left[\begin{array}{cc|c}1+i&-1&i\\0&1&0\end{array}\right]$$

$$\Rightarrow\left[\begin{array}{cc|c}1+i&0&i\\0&1&0\end{array}\right]\Rightarrow\left[\begin{array}{cc|c}1&0&\frac{i}{1+i}\\0&1&0\end{array}\right]$$

$z=\frac{i}{1+i}$

$w=0$

Se videre utregninger på neste side.

Oppgave 6

c) Forts. $z = \frac{i}{1+i} = \frac{i(1-i)}{(1+i)(1-i)} = \frac{i+1}{1+1} = \frac{i+1}{2} = \frac{1}{2} + \frac{1}{2}i$

$w = 0$

Svar: $z = \frac{1}{2} + \frac{1}{2}i$, $w = 0$

d) $$\left.\begin{aligned} 2z + iw + (5-3i)u &= 10 \\ 4z + 2iw + (10+2i)u &= 20+16i \\ 2iz - w + (4+6i)u &= 2+12i \end{aligned}\right\} \left[\begin{array}{ccc|c} 2 & i & 5-3i & 10 \\ 4 & 2i & 10+2i & 20+16i \\ 2i & -1 & 4+6i & 2+12i \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & i & 5-3i & 10 \\ 2 & i & 5+i & 10+8i \\ 2i & -1 & 4+6i & 2+12i \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & i & 5-3i & 10 \\ 0 & 0 & 4i & 8i \\ 0 & 0 & 1+i & 2+2i \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & i & 5-3i & 10 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & i & 5-3i & 10 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \underset{\Rightarrow}{I - II(5-3i)}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & i & 0 & 6i \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{i}{2} & 0 & 3i \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} z = 3i - \frac{i}{2}w \\ u = 2 \\ w \text{ er ukjent.} \end{array}$$

har $w = t$:

$$\begin{bmatrix} z \\ w \\ u \end{bmatrix} = \begin{bmatrix} 3i - \frac{i}{2}t \\ t \\ 2 \end{bmatrix} = \begin{bmatrix} 3i \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} -\frac{i}{2} \\ 1 \\ 0 \end{bmatrix} \cdot t$$

Perfekt øving, fortsett sånn!